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Modeling of Air-to-Air Visual Acquisition

A mathematical model of air-to-air visual acquisition has been developed and validated in a series of flight tests at Lincoln Laboratory. The model describes the visual acquisition process as a nonhomogeneous Poisson process in which the probability of visual acquisition per unit of time is proportional to the solid angle subtended by the target. The model has proven useful in the investigation of actual midair collisions, as well as in the evaluation of collision avoidance systems.

The Cerritos Midair Collision

August 31, 1986 was a typical warm, sunny summer day in southern California. At 11 a.m., three people boarded a single-engine Piper Archer airplane at the Torrance, Calif., airfield and departed on a flight to the resort area of Big Bear, Calif. The pilot of the Piper was William Kramer, a 53-year-old metallurgist who owned the aircraft. On board were his daughter and his wife. The Kramers planned a day of vacation at Big Bear and a return flight to Torrance later that evening. As the Piper was climbing out from Torrance, Aeromexico flight 498, a DC-9 aircraft, was descending into Los Angeles on the last leg of a regularly scheduled flight that had originated in Mexico City. In the radar control facility at Los Angeles International Airport, air traffic controller Walter White cleared Aeromexico to descend for the approach. No one seemed to suspect that anything was amiss until, at 6,500 feet, the Piper collided with the tail of the DC-9 (Fig. 1). The Kramers were fatally injured in the collision; the Piper spun out of control and slammed into a school yard in the suburban town of Cerritos. The DC-9 plunged to earth in a residential area, killing all 64 persons on board together with 15 people on the ground (Fig. 2).

In the months that followed, investigators from the National Transportation Safety Board (NTSB) carefully reconstructed the events that led to the tragedy. The wreckage was reassembled, measured, and photographed. The cockpit voice recorder was recovered and painstakingly transcribed at the NTSB laboratory in Washington, D.C. Air traffic personnel, airline officials, eyewitnesses, friends, and relatives of the pilots were interviewed. Among the many important questions to answer was a familiar one to the NTSB investigators: why didn't the two aircraft see each other in time to avoid a collision?

In pursuit of the visual acquisition question, NTSB investigator Malcolm Brenner contacted Lincoln Laboratory about studies conducted as part of Lincoln Laboratory's collision avoidance work. Lincoln Laboratory had developed a mathematical model of visual acquisition and had just completed a series of flight tests directly applicable to Cerritos. Lincoln agreed to support the NTSB, and the details of the Cerritos accident were soon being analyzed with the Lincoln model.

Origin of the Model

Visual acquisition is not a deterministic process; it is described by models that are probabilistic in nature. For small targets, acquisition takes place only when the direction of the pilot's visual search is aligned almost exactly with the line of sight to the target. The randomness of the search process, combined with the randomness of the detection probability at a given angular offset, causes acquisition times to vary greatly, even under identical visual conditions. An aircraft on a collision course is usually well above the visual resolution threshold of the eye before

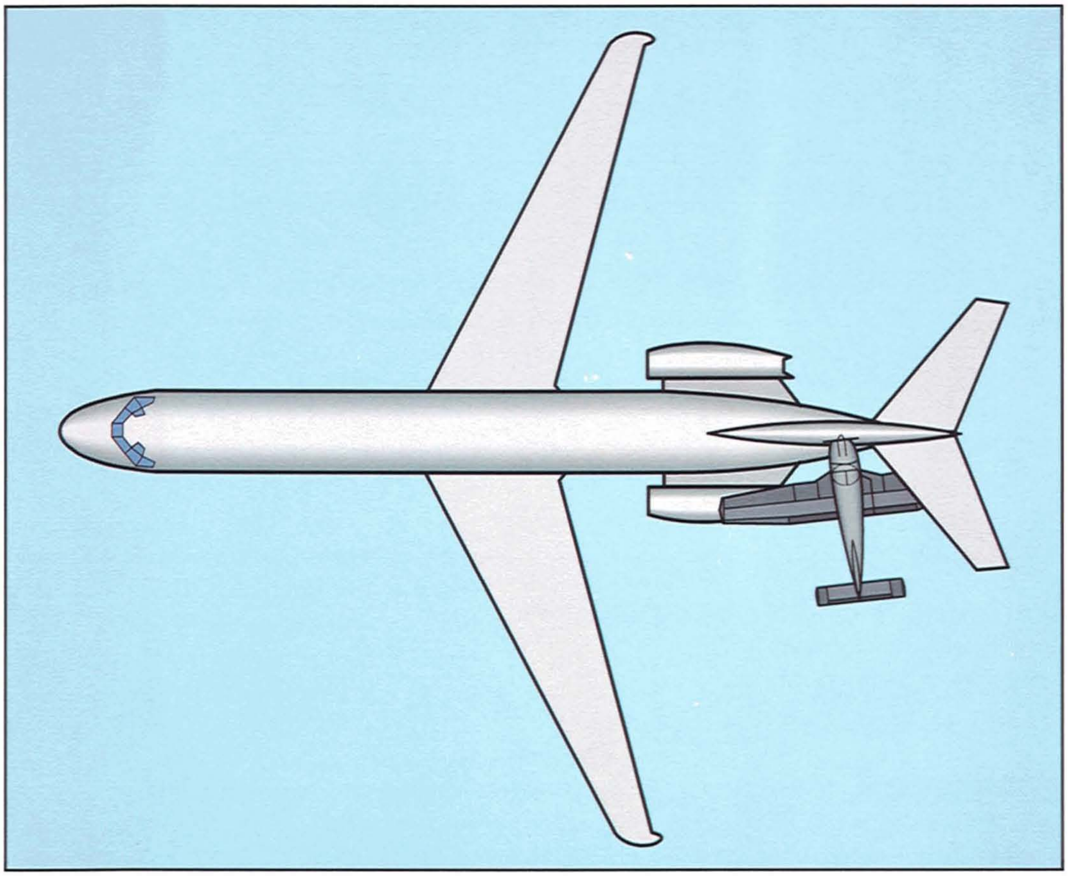

Fig. 1—Impact geometry for the Cerritos, Calif., midair collision between an Aeromexico DC-9 and a Piper Archer, on August 31, 1986. According to eyewitnesses, neither aircraft took any evasive action before impact.

it is acquired. Pilots have described the phenomenon of acquisition well above the threshold by saying that targets seem to "pop up out of the sky." In reality, the targets are visible for some time before the search results in acquisition.

The Lincoln Laboratory model originated during testing of a ground-based collision avoidance system known as the Automatic Traffic Advisory and Collision Avoidance System. During the flight tests, subject pilots flew near-collision encounters in two Lincoln Laboratory aircraft based at L.G. Hanscom Field in Bedford, Mass. Radar measured the paths of the aircraft, and radio transmission determined the time when the pilots visually acquired each other. By correlating the speeds and distances with the times of visual acquisition, the flight tests produced quantitative measures of pilot visual acquisition performance.

The immediate question to be answered was the degree of benefit from the automatic traffic advisories provided by the collision avoidance equipment. At first glance, it might seem that the benefits could be characterized by some simple statistic such as the range at which the approaching traffic was seen. But two difficulties arose. The first problem was that the visual acquisition process was highly variable and thus required a suitable statistical approach. The second and more significant problem was that many factors affected the difficulty of visual acquisition. Among these factors were the speed and geometry of approach, the size of the target aircraft, the timing and accuracy of traffic advisories, and the prevailing atmospheric visibility (visual range). How could visual acquisition data collected at one set of speeds against a single aircraft be applied to the general problem

Fig. 2—Aftermath of the Cerritos, Calif., midair collision. Fifteen people on the ground were killed in addition to 64 people on the airplane. The accident occurred on a clear day when the reported visual range was 14 nmi.

involving a range of speeds and a variety of aircraft? If the conclusions were to be generally applicable, it would be necessary to devise a means of extrapolating experimental results to other conditions. The mathematical model of visual acquisition was developed to serve this purpose.

The first step in developing the model was to establish a mathematical formalism that could serve as a powerful yet general basis for analysis. The key to such a formalism proved to be the use of the instantaneous visual acquisition rate. At any given instant, the visual acquisition rate λ can be defined as follows:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{P(\text{acq in } \Delta t)}{\Delta t}. \quad (1)$$

This rate is the probability of visual acquisition per instant time. If $\lambda = 0.02/\text{s}$, for example, then the probability of visual acquisition in one second of search is approximately 2%. The cumulative probability of visual acquisition is obtained by evaluating the acquisition probabilities for each instant as the target aircraft approaches. If λ were constant, then the result would be a set of simple equations that correspond to the classic Poisson process, a process used widely to describe physical processes such as the rate of fission of radioactive isotopes. Unfortunately, the air-to-air visual acquisition process is complicated because λ is not constant with time. In fact, λ increases steadily as two aircraft on a collision course approach closer together. The cumulative probability of visual acquisition must then be written

$$\text{P}(\text{acq by } t_2) = 1 - \exp\left(-\int_{-\infty}^{t_2} \lambda(t)\,dt\right). \quad (2)$$

This equation describes a nonhomogeneous Poisson process and provides a general mathematical formulation that can be used to describe many different visual search situations.

To apply Eq. 2, an expression for λ must be found. Several sets of test data were examined in addition to the Lincoln Laboratory data. The other sets of data included a classic flight test conducted in DC-3s by Wayne Howell for the Civil Aeronautics Administration in 1957 [2], and data collected by Control Data Corporation during a series of air-to-air photographic missions in 1973 [3]. When properly analyzed, all data sets supported the conclusion that the visual acquisition rate is proportional to the angular size of the visual target. That is,

$$\lambda = \beta \frac{A}{r^2} \quad (3)$$

where A is the visual area presented by the target aircraft, r is the range to the target, and β is a constant to be determined.

The above expression is suitable only when the atmosphere is sufficiently clear at the ranges of interest. In fog or haze, the contrast of targets can decrease rapidly with range; this dependency must be included if β is to be modeled as a constant. Laboratory experiments show that the detectability of a static target is actually determined by the *product* of the target size and target contrast. This recognition suggests that the size-contrast product should be substituted for the size expression in Eq. 3. A relationship known as Koschmieder's law states that, in a homogeneous atmosphere, apparent contrast degrades exponentially with visual range. Thus, if C_0 is the inherent contrast of the target with its background, then at range r the apparent contrast is reduced to

$$C_0 \exp\left(-2.996 \frac{r}{R}\right).$$

The constant –2.996 arises from the definition of visual range as the range at which apparent contrast is reduced to 5% of the inherent contrast.

The resulting expression for λ is

$$\lambda = \beta \frac{A}{r^2} \exp\left(-2.996 \frac{r}{R}\right) \tag{4}$$

where A is the visual area presented by the target aircraft, r is the range of the target, and R is the visual range. Here the inherent contrast is included in the value of β.

Numerical integration is usually required to find the cumulative probability of visual acquisition given by Eq. 2. However, in certain special cases a simple closed-form solution can be obtained. For example, consider the common case in which the following conditions apply: (*a*) the visual range is so large that it plays no role in degrading contrast, (*b*) the target aircraft is on an unaccelerated collision course, and (*c*) the value of β and A are constant. Condition *a* allows us to set R to infinity. Condition *b* allows us to write the range to the target as $r = -\dot{r}t$ where $\dot{r}$ is the closing rate and t is the time to collision. Condition *c* allows the integration to be carried out with the following result:

$$P(\text{acq by } t_2) = 1 - \exp\left(\frac{\beta A}{\dot{r}^2 t_2}\right). \tag{5}$$

The Value of Traffic Advisories

Air traffic controllers assist visual acquisition by providing traffic advisories for nearby aircraft detected by radar. Such an advisory might be "American 578, you have traffic at two o'clock, three miles, reporting seven thousand feet." These advisories are issued on a "workload permitting" basis, which means that in some busy situations the air traffic controller may curtail the service. Automatic traffic advisories are generated by the Traffic Alert and Collision Avoidance System, or TCAS (see "TCAS: A System for Preventing Midair Collision" on page 437 in this issue). These advisories are displayed on a plan-position display located on the aircraft instrument panel.

Traffic advisories improve visual acquisition in two principal ways. First, they increase the fraction of the crew's time devoted to the visual search for traffic. Second, they concentrate visual search in the proper direction. A quantitative measure of crew performance with such advisories was an objective of Lincoln Laboratory's work in TCAS flight testing. Lincoln Laboratory built and installed a prototype TCAS in a Cessna 421 aircraft; the TCAS provided traffic advisories through a special interface to a weather radar CRT. The bearing accuracy of these advisories was approximately 7° (one standard deviation) and the position of the traffic was updated once per second. Six different subject pilots flew a variety of missions that resulted in data for 66 near-miss encounters.

The subjects were aware that, from time to time, a second test aircraft would conduct an intercept that would trigger the collision avoidance alarms. The interceptor was vectored by ground radar to achieve a near-zero horizontal miss distance with an altitude separation of 200 to 500 ft. From the viewpoint of the subject pilots, the intercepts presented visual acquisition problems similar to those of actual collision situations. When the subjects used the TCAS, they visually acquired in 57 of the 66 encounters. The median range of visual acquisition was 1.4 nmi. In five of the nine cases of acquisition failure, the subject aircraft received a *climb* advisory from the TCAS and entered a nose-up attitude that prevented acquisition of the intruder passing below.

To understand these results fully, we analyzed the data in accordance with the visual acquisition model. A robust estimation technique was developed to obtain the value of β. The technique was based on the fact that, according to the model, the probability of visual acquisition within any time interval for which β is constant can be determined from the time integral Q of the solid angle-contrast product of the target. Based on Eqs. 2 and 4, Q can be defined as follows:

$$P(\text{acq by } t) = 1 - \exp[-\beta Q(t)] \qquad (6)$$

where

$$Q(t) = \int_{-\infty}^{t} \frac{A}{r^2} \exp\left(-2.996 \frac{r}{R}\right) dt. \qquad (7)$$

Since Q represents the opportunity for visual acquisition that the target has provided, Q is called the *opportunity integral*. The estimated value of β is obtained by plotting observed acquisition probability as a function of Q and then finding the value of β that, when inserted in Eq. 6, best fits the plotted data. Figure 3 shows the acquisition probabilities for TCAS flight test data plotted as a function of Q. The value of β that best fits this curve is 140,000/ster-s.

A numerical example places the value of β in perspective. Consider the acquisition rate for a Boeing 727 jet transport when seen head-on at a range of 3 nmi. The visual area of the aircraft is about 430 sq ft when seen from this angle. At a range of 3 nmi, the aircraft will subtend an angle α of 1.3×10^{-6} ster. Thus the rate of visual acquisition is $\lambda = \beta\alpha = 0.18/\text{s}$, or approximately 18% per each second of alerted search. Of course, the angular size of the aircraft is constantly changing, and the integration indicated in Eq. 2 must be carried out in order to compute the cumulative probability of visual acquisition over a period of time.

Unalerted-Search Flight Tests

During the experiments with collision avoidance equipment, the focus of testing had been on visual acquisition aided by automatic traffic advisory avionics. Because most Visual Flight Rule flights have no traffic advisory services, a need remained to establish a baseline for unalerted search. With the support of the FAA, a new series of test flights were initiated to provide this baseline. In these tests, a group of 24 general aviation pilots each flew a Beech Bonanza on a triangular cross-country flight of about 45 minutes' duration. During this time, a Cessna 421 aircraft conducted three intercepts; it flew both over and under the test route to provide a target for visual acquisition.

Even though the subjects had no traffic advisories to alert them to the time and direction of approach of the intercepts, additional care was taken to prevent undue concentration on visual search. The tests were conducted as a general study of single-pilot cockpit technique. Initial subject briefings stated that the experimenters were interested in seeing each pilot's individual technique, that there was no single standard for right and wrong, and that the subject should fly in as normal and relaxed a manner as possible. Data were collected in the cockpit by a safety pilot. Periodically this pilot would ask the subjects questions such as "Where is Worcester Airport from our present location?" At several points during the flight, the safety pilot would ask subjects to rate their workload on a 1-to-9 scale. They were also asked to call out all traffic when it was first seen; the times of sightings were carefully recorded for later analysis. The subjects were told that other company traffic might be operating in the area, but that the Lincoln Laboratory control center would make sure that this traffic knew their location and kept clear of their altitude.

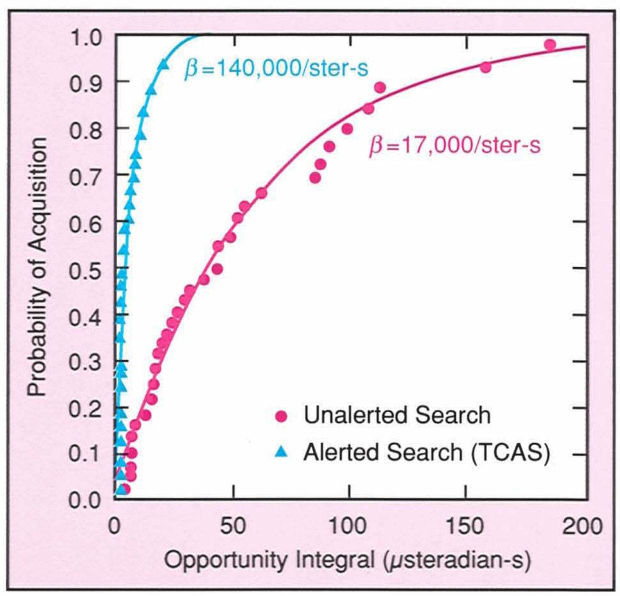

Fig. 3—Measured and modeled visual acquisition probabilities. The probability is defined in terms of the opportunity integral—a measure of the angular size of the visual target and the duration of the search.

The test design appeared to be highly successful in preventing subjects from concentrating unduly on visual search. Only two subjects guessed that the Cessna 421 was the "company traffic," and in neither case did this insight arise before the last intercept was over.

Data were obtained for 64 encounters. Visual acquisition was achieved in only 36 of these encounters (56% of the total), and the median acquisition range for the 36 encounters was 0.99 nmi. Figure 3 provides a plot of the acquisition probability versus opportunity integral for these unalerted-search data. The value of β that best fits the data is approximately 17,000/ster-s.

In a comparison between the unalerted-search results and the alerted-search results from the earlier TCAS tests, the pilots who used the TCAS traffic advisories exhibited a β value that was higher by a factor of 8.2. The higher factor implies that the presence of the TCAS traffic advisory increased search effectiveness by a factor of 8. In other words, one second of search with the TCAS advisory was as effective as eight seconds of search with no alert.

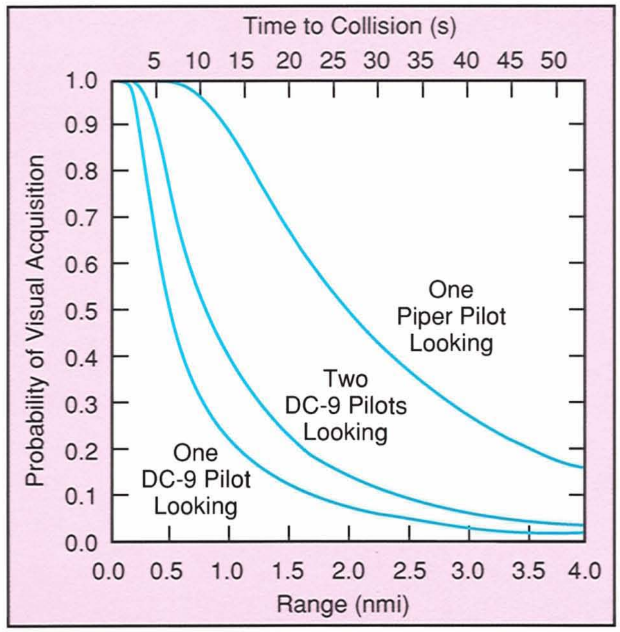

Fig. 4—Model prediction of visual acquisition probability as a function of separation between aircraft for the Cerritos scenario. These curves assume unalerted visual search.

Analysis of the Cerritos Midair Collision

Because neither aircraft in the Cerritos accident had received a traffic advisory, the results of the unalerted-search flight tests were crucial to the analysis of that accident. Data provided by the NTSB were used to determine the closing rate between the aircraft (271 knots) and the visual areas of each aircraft as seen from the other (72 sq ft for the Piper and 588 sq ft for the DC-9). Figure 4 shows the results of the analysis. The bottom two curves describe the search for the Piper from the DC-9. The two-pilot curve is derived by assuming that the effect of the second pilot is to double the search performance for the aircraft. The figure shows that the probability of visual acquisition is small until the last 15 seconds or so before collision. The probability of effective collision avoidance (using 12-s warning time as a criterion) is only 42%, even assuming that both pilots are searching.

The top curve describes search for the DC-9 from the Piper. The acquisition probability increases significantly over the last minute of flight. At 12 s before collision, the probability of visual acquisition is 90%. Acquisition of the DC-9 is easier because it provides a visual target that is about eight times larger than the Piper.

This analysis shows that only in the last minute prior to collision does a significant probability for visual acquisition exist. Events that occurred earlier (such as traffic advisories or cockpit distractions) are unlikely to have had any effect on the failure to acquire.

A second significant result was to focus attention on why the pilot of the Piper did not acquire. The results of the model apply only if normal visual search is underway. Therefore, an unusual factor must have existed that prevented normal search on board the Piper. In fact, the NTSB investigation revealed evidence that pilot Kramer, who was navigating by visual ground references, had misidentified a freeway and had strayed from his intended course. At the time of the accident, he would probably have been looking out the left side of the aircraft for Disneyland, a prominent visual landmark used by local pilots. If this idea was correct, it might explain why the DC-9, approaching from 50° to the right of the nose of the aircraft, would not have been seen.

In the search for ways to prevent similar accidents in the future, NTSB posed the following question: would the Cerritos accident have been prevented if the Aeromexico DC-9 had been equipped with a TCAS? Because the Piper was not equipped with an altitude-reporting transponder, the TCAS would have been unable to issue a resolution advisory. It would, however, have issued a traffic advisory that would have accurately depicted the angular position and range of the Piper. A second set of calculations were carried out under the assumption of alerted search by the DC-9 crew. These calculations showed that if they had been alerted by a TCAS traffic advisory, the crew would have had a 95% chance of seeing the Piper in time to avoid. The NTSB final report concluded that "had flight 498 been equipped with a TCAS, the accident might not have occurred." They recommended that the FAA "expedite the development, operational evaluation, and final certification of the Traffic Alert and Collision Avoidance System for installation and use in certificated air carrier aircraft [3]."

Conclusion

Maintenance of safe separation between aircraft is often dependent on the ability of pilots to see and avoid other traffic. But even when weather conditions are favorable for visual separation, midair collisions often occur without the crews of either colliding aircraft seeing the other. This fact raises central questions concerning the performance that can be expected of air crews and the potential benefits of services such as the automatic traffic advisories available from collision avoidance systems.

Although visual acquisition is a complex process, a useful statistical model of pilot performance has been developed at Lincoln Laboratory. The model explains a variety of experimental results that, without a common framework for analysis, would be difficult to compare. The model is currently used by the FAA to evaluate collision avoidance systems, airspace regulations, and waiver requests. The NTSB has also used it to provide insight into crew performance in accident scenarios. The model has a significant limitation: it can be applied only where the pilot performance level can be assumed to approximate that for which flight test data are available. Extension of the model to other situations (such as high-workload landing phase or military training situations) may require pilot performance testing with either actual flight or realistic flight simulators.

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