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Using Langmuir Probes to Measure Ion Velocities in Pyrotechnic Plasmas

The very fluctuations that often complicate measurements of plasma parameters can be used to measure ion velocities in pyrotechnic plasmas. When burned in a vacuum, many pyrotechnic devices will form exhaust plumes that are hot enough to contain small quantities of singly ionized atoms. This pyrotechnic plasma can be maintained in the laboratory for only a relatively short period of time because the chamber fills within several seconds, eliminating the desired low-pressure environment. Furthermore, the fluctuating intensity of the burning pyrotechnic causes correspondingly large fluctuations in the plasma. These fluctuations make dependable determinations of plasma parameter values extremely difficult. Under these conditions an old standby of plasma physics — the Langmuir electrical probe — can provide surprisingly reliable estimates of the average ion velocity in the pyrotechnic exhaust plume [1]. The variation in the pyrotechnic's rate of combustion creates concentrations of ions. These concentrations, or ion clumps, are tracked as they escape the reaction chamber; by measuring the time it takes one of these clumps to traverse a known distance, the ion velocity within the plume can be estimated.

Pyrotechnics have been used on the battlefield for over seven centuries. The application of pyrotechnics to countermeasures is much more recent, but is an important part of modern warfare. In the tactical arena, pyrotechnics are often used for deception. Combat aircraft are equipped with infrared flares to decoy heat-seeking missiles. The flares are ejected from the aircraft to confuse missile seekers during their terminal homing phase. Pyrotechnics are also being considered for countermeasure applications in strategic systems. Consequently, the physical characteristics of pyrotechnic reactions need to be well understood, and so an experimental and theoretical modeling activity is in progress.

Many times pyrotechnic flares will, when burned in a vacuum, form exhaust plumes hot enough to contain small quantities of singly ionized atoms. Unfortunately, fluctuations in the burn rate of the flare will often frustrate attempts at plasma measurements. We have found, however, that by taking advantage of these fluctuations we can estimate ion velocities inside a pyrotechnic exhaust plume.

We placed two Langmuir probes in a vacuum chamber. (An explanation of this technique appears in the box, "The Langmuir Probe.") The first one was 20 cm above the reaction chamber (this separation was used to collimate ions that exited from the combustion chamber and entered the vacuum chamber); the second probe was placed 4 cm above the first. The exiting ions established currents in each probe and fluctuations in the pyrotechnic burn rate gave those currents' waveforms a random but correlatable shape; that is, the current in each probe varied randomly with the fluctuations in the chemical reaction but, since the variations in both currents responded to the same chemical event, features within each waveform could be correlated with each other. By taking the crosscovariance (see analysis following Eq. 1) of the two currents' waveforms, the time required for the ions to transit the 4 cm between the probes was determined. And, from the transit time, the ion velocity was estimated.

Experimental Data

Figure 2 shows the basic experimental setup. A pyrotechnic plasma was generated in-

Fig. 1 — Plume from a pyrotechnic device reacting inside a bell-jar vacuum chamber.

side a combustion chamber using the chemical reaction

$$3\ Zr + 2\ CsNO_3 \rightarrow 3\ ZrO_2 + 2\ Cs + N_2 .$$

This reaction has a combustion temperature of 3,300 K, and in any given burn we expect several percent of the cesium atoms that have been expelled from the combustion chamber to have lost valence electrons. These Cs^+ ions travel upwards and strike two cylindrical Langmuir probes held at a bias voltage of −3 V with respect to the combustion chamber. No magnetic fields are present and the ambient pressure outside the combustion chamber is 0.05 to 0.1 Torr. The negative voltage bias ensures that only the Cs^+ ions hit the probes' conducting surfaces. The ions absorb conduction electrons in the probes, thereby generating milliamp currents inside the probe circuitry. The amplitude of this current typically varies on a time scale of 10 to 100 μs, depending on the rate of the chemical reaction.

The cause of the fluctuation is not known, but it does appear to be associated with an intrinsic variability in the rate of chemical reaction, not with the construction of the combustion chamber. The variability in the chemical reaction produces varying amounts of Cs^+ ions. Consequently, the ions in the exhaust plume outside the combustion chamber travel upward in clumps rather than in a smooth distribution. As each ion clump passes a probe it generates a

The Langmuir Probe

A Langmuir probe is a small, metallic electrode that is in contact with a plasma and is connected to circuitry that 1) imposes upon the probe an adjustable voltage bias with respect to ground, and 2) measures the current through the electrode generated by the plasma's response to the voltage bias. Langmuir probes are used to measure local conditions inside a plasma. Other measurement techniques — although perhaps more accurate — tend to provide only global results. The advantage of a Langmuir probe, however, is somewhat offset by the major pitfall of using the probe; the probe itself can create local variations in the surrounding plasma. As a result, great care must be taken in collecting and interpreting data from Langmuir probes. While they are not used as often in the laboratory today as they were in the past, Langmuir probes have, to a certain extent because of their simplicity and ruggedness, become the instrument of choice for spacecraft experiments that measure plasmas outside the earth's atmosphere.

Although this sort of electrical probe was used by others in investigations of gas discharges prior to his use, Irving Langmuir (Nobel prize, chemistry, 1932) and his coworkers set forth a theory of operation for the probes in the mid-1920s that made them one of the most powerful tools in plasma diagnostics. He introduced the term "plasma" to describe the electrically neutral collections of ionized particles he was studying and he derived a set of equations that are still used today (sometimes in modified form) to describe the performance of Langmuir probes in the presence of low-temperature, low-density plasmas.

temporary surge in the probe current. We can use the crosscovariance of the random fluctuations in the upper and lower probe currents to determine the lag time, T_p, marking the peak in the crosscovariance function. Because it marks the peak, T_p must be the characteristic lag time between the appearance of a clump at the first probe and the appearance of the same clump at the second probe. Since the probes are vertically separated by 4 cm, the average Cs^+ ion velocity inside the pyrotechnic exhaust plume can be estimated by dividing T_p into 4 cm.

We used a transient waveform analyzer to measure and store data from the probes. The photograph in Fig. 3 shows two simultaneous oscilloscope traces, which record current from the lower probe (signal *A*) and from the upper probe (signal *B*). Four sets of data were obtained from three separate burns. Each data set represents a total elapsed time of 15 ms, sampled at 1-μs intervals.

Although the two traces are not identical, the major features of signal *B* have been foreshadowed by signal *A*. Rather than simply measure the delay between these features, we used the

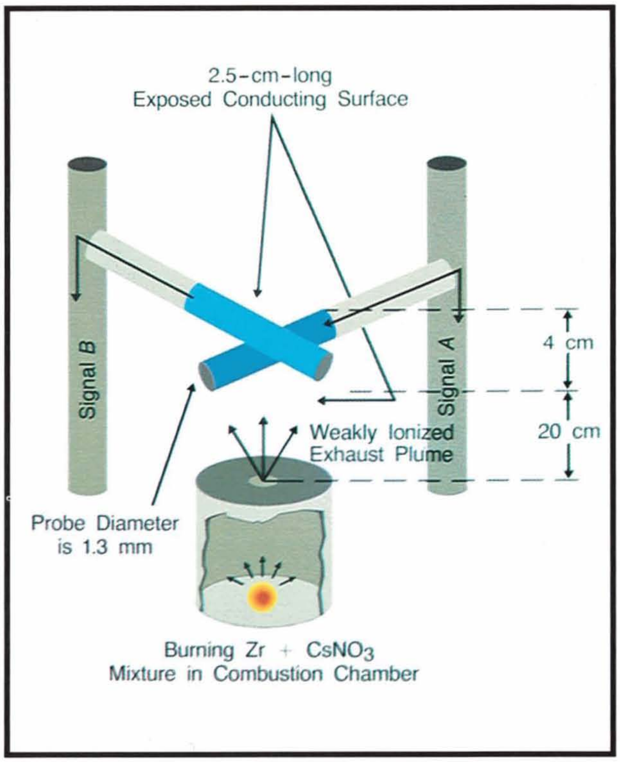

Fig. 2 — This is the experimental setup used to determine the velocity of ions in a pyrotechnic plasma.

crosscovariance relationship between the two probes' data to find the characteristic delay (T_p) associated with the passage of each of these features between the probes. In doing so we define A_i, B_i to be the sample value of signals A and B taken at a time $i\,\Delta t$ from the start of a given data set (where $\Delta t = 1\ \mu s$). Hence A_0, B_0 are the first samples of the data set; A_1, B_1 are the samples taken one microsecond later; and so on to the last two samples of the data set $A_{14,999}$, $B_{14,999}$.

For each data set we define the crosscovariance function C_j to be

$$C_j = \begin{cases} \dfrac{1}{N-|j|-1} \displaystyle\sum_{i=j}^{N-1} (A_{i-j} - \langle A\rangle)\cdot(B_i - \langle B\rangle) & \text{for } j \geq 0 \\[2ex] \dfrac{1}{N-|j|-1} \displaystyle\sum_{i=0}^{N-|j|-1} (A_{i+|j|} - \langle A\rangle)\cdot(B_i - \langle B\rangle) & \text{for } j < 0 \end{cases} \qquad (1)$$

where

$$N = 15{,}000\ ;$$
$$j = -450, -449, \cdots, -1, 0, 1, \cdots, 449, 450\ ;$$
$$\langle A\rangle = \frac{1}{N}\sum_{i=0}^{N-1} A_i\ ;$$
$$\langle B\rangle = \frac{1}{N}\sum_{i=0}^{N-1} B_i\ .$$

The crosscovariance defined in Eq. 1 measures the similarity between series A and series B. As N gets very large, the crosscovariance approaches zero if A and B have no intrinsic similarity. The coefficient j represents a shift of the A signal against the B signal by $j\ \mu s$. As j takes on values from −450 to +450, the A signal is first shifted to the left by 450 μs, at which point a crosscovariance value is computed. Then the crosscovariance is computed for a left shift of 449 μs, and so on until the crosscovariance is found for shifts of 450 μs to the left, 450 μs to the right, and all points in between. The shift that has the largest value of crosscovariance associated with it determines the value of T_p. In Fig. 3, for example, signal A will match signal B most closely (the value of the crosscovariance function will reach a maximum) if it is shifted to the right by about 80 μs. When this match occurs, we can regard the fluctuations in signal A as predicting the presence of fluctuations in signal B about 80 μs later, because A was shifted to the right, or to a later point in time, by 80 μs. The crosscovariance function C_j has been calculated for negative as well as positive values of j. The crosscovariance function for negative values of j provides an estimate of the noise level in the measurements. Since signal A is from the lower probe, a fluctuation in signal A should precede a fluctuation in signal B and therefore the peak in C_j should occur for a positive value of index j.

Figure 4 is a graph of C_j vs. j for the crosscovariance function from data set 3. There is a well-defined peak at $j = 35$, which corresponds to a T_p value of 35 μs. Table 1 shows the T_p values, their average, and the associated standard deviations of the crosscovariance function for the four 15,000-point data sets analyzed in these experiments.

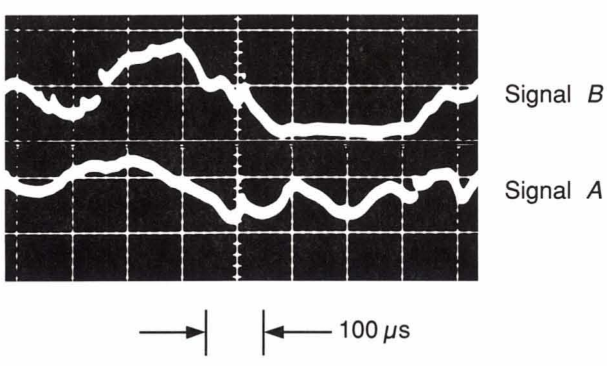

Fig. 3 — Oscilloscope traces of the upper (signal B) and lower (signal A) probes immersed in the plasma.

Analysis

A simple theoretical model of the ion clumps in the pyrotechnic exhaust plume supports the 37-μs time lag given by the data. Consider a distribution of ions moving along the x- (vertical) axis of a vacuum chamber. The probability of an ion selected at random having a velocity between v and $v + dv$ is defined as $g(v)\,dv$. We define $N(x,t)\,dx$ to be the number of ions located between x and $x + dx$ at a time $t \geq 0$. The lower probe, which generates signal A, is located at $x = 0$ and

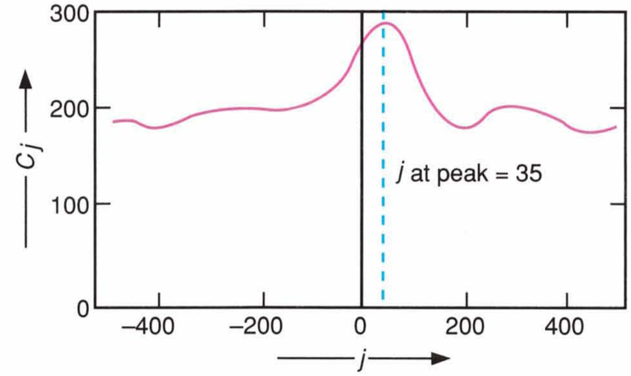

Fig 4. — This crosscovariance function, from data set 3, shows that the peak of the function corresponds to a delay of 35 μs between the probes. Therefore, the two signals are most similar when signal A is shifted to the right by 35 μs.

the upper probe, which generates signal B, is at $x = x_0$. At $t = 0$, a Gaussian-shaped ion clump of standard deviation (width), σ, is centered on the lower probe

$$N(x,0) = N_0 e^{-(x^2/2\sigma^2)} \tag{2}$$

where N_0 is an undetermined constant with the units of ions per unit distance. The current generated in the upper probe at a time $t > 0$ is proportional to

$$N(x_0, t) = \int_0^\infty N(x_0 - vt, 0) g(v)\, dv. \tag{3}$$

Assuming that the ion-velocity distribution inside the combustion chamber is reasonably close to statistical equilibrium and thus is Maxwellian in nature, we can approximate the exhaust plume as a molecular beam [2] and set

$$g(v) = \frac{4v^2}{\sqrt{\pi}\alpha^3} e^{-v^2/\alpha^2} \tag{4}$$

where

$\alpha = (2kT_c/m)^{1/2} = 640$ m/s
k = Boltzmann's constant $= 1.38 \times 10^{-23}$ J/K
m = mass of cesium ion $= 2.2 \times 10^{-25}$ kg
T_c = combustion temperature of zirconium-cesium nitrate reaction (3,300 K).

The value of $\alpha = 640$ m/s gives an order of magnitude indication of the ions' speed, and, since the current fluctuations occur on a time scale of 10 to 100 μs (see Fig. 3), we conclude that the one-standard-deviation width (σ values) of typical ion clumps lies between $\sigma = 6.4$ cm and $\sigma = 0.64$ cm. Substituting Eq. 4 into Eq. 3 and integrating gives us

$$N(x_0, t) = \frac{4N_0}{\alpha^3 B^3} e^{-(x_0^2/2\sigma^2)} \times \left\{ \frac{A}{2\sqrt{\pi}} + \frac{1}{2} e^{A^2} \left[1 + \operatorname{erf}(A)\right] \left[A^2 + \frac{1}{2}\right] \right\}$$

where

$$B = \sqrt{\frac{1}{\alpha^2} + \frac{t^2}{2\sigma^2}},$$

$$A = \frac{x_0 t}{2\sigma^2 B}$$

We set x_0 equal to 4 cm, the distance between the upper and lower probes; t_{max} is defined as the time at which $N(x_0, t)$ increases to a maximum as t increases from zero.

The value of t_{max} is a relatively insensitive function of σ. When σ is 6.4 cm, $t_{max} = 47$ μs. As σ decreases, the value of t_{max} increases, to $t_{max} = 48$ μs at $\sigma = 3.6$ cm and $t_{max} = 50$ μs at $\sigma = 0.64$ cm. Adopting the standard deviation of the four data points as the uncertainty in the measured value of t_{max}, the t_{max} values for $\sigma > 3.6$ cm agree with the average T_p value of 37 ± 11 μs given by the data in Table 1. We can, therefore, tentatively conclude that T_p can be used to estimate the average ion velocity inside the pyrotechnic plasma.

Table 1. Measured Lag Times

Data Set	T_p *(μs)*	*Estimated Ion Velocity (m/s)*
1	37	1,081
2	52	769
3	35	1,143
4	25	1,600
Average Value ± Standard Deviation	37 ± 11 μs	1,148 ± 343 m/s

Conclusions

The fluctuating nature of weakly ionized, pyrotechnic plasmas often makes it difficult to use the traditional methods of plasma physics to analyze a pyrotechnic exhaust plume with any degree of precision. However, it is possible to use these fluctuations to advantage instead of regarding them as just another source of experimental error. When two Langmuir probes are set up inside an exhaust plume so that the ion clumps traveling up the plume impinge on one probe before the other, the T_p lag time between the two probes' fluctuating currents can be used to estimate an average velocity for the ions in the pyrotechnic plasma. The experiment is easy to perform, the data analysis is straightforward, and the final measurements agree with theoretical expectations.

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