



30°) that we shall actually search in. Denote this search space by  $J$ .

One searches using a telescope with a finite field of view. In practice we always examine an entire field, never a fraction of a field nor more than one field at a time. Hence the search space  $J$  consists of a discrete set of fields of view. Number these by the index  $j = 1, 2, \dots$ . In particular, since the celestial sphere encompasses  $4\pi$  sr,  $\max(j) < \infty$ .

Before the minor planet is found one assigns an a priori *target distribution* on the search space  $J$ ,  $p: J \rightarrow [0,1]$  (the notation means that  $p$  is a function defined over the set  $J$  which maps elements of  $J$  into the domain zero to unity inclusive). The target distribution is the a priori probability of finding an asteroid in field of view  $j \in J$  before one starts the search. For main belt asteroids a reasonable model for  $p$  is  $p$  is uniform over all heliocentric ecliptic longitudes and over the heliocentric ecliptic latitude range  $<10^\circ$  (or  $5^\circ$  or  $20^\circ$ ). For Earth-approaching minor planets, both because of parallax effects and the inherent spread of Earth-approaching asteroids over orbital element space (particularly in inclination), a reasonable model for  $P$  is that  $P$  is uniform over the topocentric celestial sphere. In any case

$$\sum_{j \in J} p(j) \leq 1.$$

When one examines a field of view (whether a video frame or an exposed plate) for an asteroid one expends a certain amount of effort trying to detect the asteroid. In the photographic case one is looking for the streak that the moving minor planet has left. In the video mode one looks for two displaced dots from frames taken at different times (and after the stars have been electronically subtracted). One may look at the same field of view several times. The cost of performing  $k$  inspections in the  $j$ th field of view is measured by a *cost function*  $c(j,k): J \times \{0,1,2, \dots\} \rightarrow [0,\infty]$ . Clearly  $c(j,0) = 0 \forall j \in J$  (no effort implies no cost).

One could measure cost by the time spent examining a field of view plus the time spent in moving to the next field of view (this makes  $c$  nonlocal and is not desirable). Operationally we always spend the same time in each field of view (more or less). Also, because  $[\text{area}(J)]^{1/2}/\text{slew speed} \ll$  time spent examining a field of view, the nonlocal element of  $c$  is both unimportant and varies little. We typically spend 45 sec examining a field, the telescope slews at  $4^\circ/\text{sec}$ , and we rarely search more than 500 square degrees per night. Hence  $[\text{area}(J)]^{1/2}/\text{slew speed} = 5.6$  sec. (For photographic searches it is a good approximation too because large plates are usually used; i.e.,  $6^\circ \times 6^\circ$ .) Thus I shall measure cost by time and, in the instance of the asteroid search, specialize to the case when the incremental cost of the  $k$ th examination in field of view number  $j$ , viz.,

$$\gamma(j,k) = c(j,k) - c(j,k-1)$$

is a constant independent of both  $j$  (i.e., the telescope is fast or the plates are large and all fields of view are treated equally) and  $k$  (e.g., the same field of view is equally well inspected each time).

When one does examine a field of view of the search space looking for an asteroid then there is a conditional probability of detecting it on or before the  $k$ th inspection of that field of view (given that it is there). This function, for field of view number  $j$  and examination  $k$ , is denoted by  $b(j,k): J \times \{0,1,2, \dots\} \rightarrow [0,1]$ . Naturally  $b(j,0) = 0 \forall j \in J$  (you cannot find it if you do not look for it). From the *detection function*  $b$  one can construct the probability of failing to detect the asteroid on the first  $k-1$  scrutinizations of field of view number  $j$  and then succeeding on the  $k$ th one (given that the asteroid is in field of view number  $j$ ); viz.,

$$\beta(j,k) = b(j,k) - b(j,k-1).$$

There is a lot of physics and mathematics subsumed in the detection function. Clearly it depends on the asteroid's apparent mag-

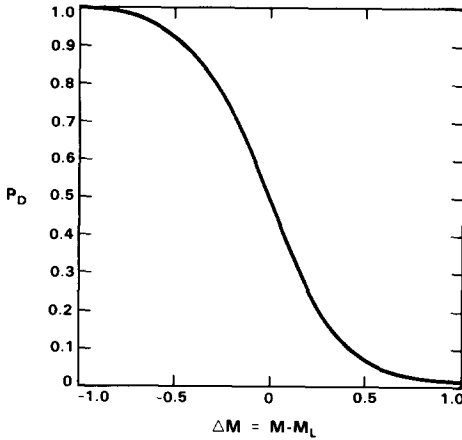


FIG. 1. Probability of detection as a function of "distance" from limiting magnitude  $m_L$ . The functional form is  $\text{constant}/(1 + \exp[5(m - m_L)])$ .

nitude, the background star density, the night sky background brightness, the resolution element size of the detector(s), the false alarm probability one is willing to accept, how tired one is, etc. Since the ecliptic is unchanging, atmospheric extinction can be computed, the Moon's position is known, etc., this is a computable function. Indeed we are developing software to realistically do so in a physically correct way. Operationally, for a fixed set of external parameters, our detection probability has the shape shown in Fig. 1 where  $m_L$  is our quoted limiting magnitude (e.g., where the probability of detection is 50%). The form shown in the diagram will be used to compute the optimal search plans given below.

Finally I need to define a *search plan*. A discrete search plan is a sequence  $\xi = (\xi_1, \xi_2, \xi_3, \dots)$  which tells the searcher to first look in cell  $\xi_1$ ; if the asteroid is found there then terminate the search but if the minor planet isn't found then look next in field of view  $\xi_2$ , etc. A global way to describe this is by a function which specifies the *allocation* of effort devoted to each field of view  $j$ . To this end, define  $f: J \rightarrow [0, \infty)$ ;  $f(j)$  is the number of examinations in field of view number  $j$ .

Above I referred to searches for a fixed target. Clearly the minor planets we are try-

ing to find are moving. Earth-approaching asteroids can have geocentric angular speeds of a degree per day (or more). I have made the assumption that even these objects are fixed when compared to our search rate. The mathematical formulation of this approximation is [area ( $J$ )/search rate]  $\cdot$  asteroid angular speed  $\ll$  field of view. For our parameters [area ( $J$ ) = 500 square degrees, search rate = 100 square degrees/hr, asteroid angular speed =  $1^\circ$ /day, field of view =  $2^\circ$ ] an asteroid could traverse only one-twelfth of a field of view before the entire search is completed. Hence the real problem fits into the formalism reasonably well. For faster moving minor planets the optimal search plans developed below need to be corrected for the asteroid's motion.

### 3. OPTIMAL SEARCHES

Given the cost of searching field of view number  $j$  a total of  $k$  times,  $c(j, k)$ , the total cost of performing the search plan  $\xi$  with allocation  $f$  is

$$C[f] = \sum_{j \in J} c(j, f(j)).$$

The total number of examinations over all fields of view is  $\sum_{j \in J} f(j)$ . Similarly the total probability of minor planet detection with this allocation of effort is  $P[f]$ ,

$$P[f] = \sum_{j \in J} p(j) b(j, f(j)),$$

where  $b(j, k)$  is the conditional probability of finding the asteroid in field of view number  $j$  after  $k$  examinations of that field of view given that it's in that field of view.

There are four types of searches one might define as optimal. One might be interested in maximizing the total probability of detection when constrained to a given number of inspections (say  $K$ ). If the incremental cost function  $\gamma(j, k) = c(j, k) - c(j, k - 1)$  is a constant, then (after a suitable renormalization) one is demanding that  $P[f]$  be a maximum for  $C[f] \leq K$ . Such a search is termed *totally optimal*. If one demanded optimality for all  $K = 1, 2, 3, \dots$ , then the

search is called *uniformly optimal*. A third type of search plan that one might consider is the search plan that maximizes the probability of detection with respect to the incremental cost and does so at every step of the search. Mathematically one finds the value of  $j$  which maximizes  $p(j)\beta(j,k)/\gamma(j,k)$  at each  $k$ . These searches are called *locally optimal*. Lastly one might entertain a search plan that minimized the total expected cost (i.e., was *the fastest*) to find the target.

The essential assumptions necessary to cast the asteroid search into the simplest form of the mathematical superstructure that Stone (1975) outlines are

(1) that the asteroid is stationary (i.e., search rate is high compared to the asteroid's angular speed),

(2) that the search space is discrete (i.e., a fixed field of view),

(3) that the allocation of effort is discrete (i.e., no favored fields of view), and

(4) that  $\gamma$  is bounded away from zero and  $p(j)\beta(j,k)/\gamma(j,k)$  is a decreasing function of  $j$  (i.e., no free examinations of a field of view and the larger the search space the more difficult to detect).

I do not believe that the physics or astronomy is strained by these strictures. In fact (5)  $\gamma = \text{constant}$  is not unreasonable (i.e., the telescope moves smartly). The important point is that under these five limitations the totally optimal search plan, the uniformly optimal search plan, the locally optimal search plan, and the fastest searches are all *identical*. Not only that, it can be explicitly exhibited. See Stone's text for the rigorous mathematical statements of the relevant theorems and their proofs.

4. THE SEARCH PLAN

I need just a bit more mathematics before I can exhibit the solution to the optimal search problem. The search plan  $\xi = (\xi_1, \xi_2, \xi_3, \dots)$  is a sequence of values  $\xi_i \in J$  for  $i = 1, 2, 3, \dots$ . These specify that the  $i$ th examination be in field of view  $\xi_i$  if the previous  $i - 1$  inspections failed to detect

the asteroid in fields of view  $\xi_1, \xi_2, \dots, \xi_{i-1}$ . Let the set of all such search plans be denoted by  $\Xi$ . Introduce the probability  $P[n, \xi]$  (and the cost  $C[n, \xi]$ ) of detecting the asteroid on or before the  $n$ th examination while performing search plan  $\xi \in \Xi$  (of the first  $n$  inspections). Finally, let  $r(j, n, \xi)$  be the number of scrutinizations out of the first  $n$  that are placed in the  $j$ th field of view while following search plan  $\xi$ . A uniformly optimal search plan [for  $\gamma(j, k) = 1$ ; this is an unimportant normalization]  $\xi^* \in \Xi$  is one such that

$$P[n, \xi^*] = \max\{P[n, \xi] : \xi \in \Xi\},$$

$$n = 1, 2, \dots, K.$$

A locally optimal search plan  $\xi^*$  is one such that  $\xi_1$  is determined by [ $\gamma \neq 0$  necessarily]

$$\frac{p(\xi_1)\beta(\xi_1, 1)}{\gamma(\xi_1, 1)} = \max_{j \in J} \frac{p(j)\beta(j, 1)}{\gamma(j, 1)}$$

and having determined the field of view for the first  $n - 1$  examinations ( $\xi_1, \xi_2, \dots, \xi_{n-1}$ ) the field of view for the  $n$ th one is determined from

$$\frac{p(i)\beta(i, r(i, n - 1, \xi) + 1)}{\gamma(i, r(i, n - 1, \xi) + 1)}$$

$$= \max_{j \in J} \frac{p(j)\beta(j, r(j, n - 1, \xi) + 1)}{\gamma(j, r(j, n - 1, \xi) + 1)}$$

with  $\xi_n^* = i$ . Now define  $k_n = r(\xi_n, n, \xi)$ . The notation means that the  $n$ th examination of the search plan  $\xi$  is placed in field of view  $\xi_n$  and that it is the  $k_n$ th time that this field of view has been searched. The average cost to find the asteroid can be expressed in a variety of ways if the limit as  $n \rightarrow \infty$  of  $P[n, \xi]$  is unity;

$$\mu(\xi) = \sum_{n=1}^{\infty} C[n, \xi](P[n, \xi] - P[n - 1, \xi])$$

$$= \sum_{n=1}^{\infty} \sum_{m=1}^n \gamma(\xi_m, k_m) p(\xi_n)\beta(\xi_n, k_n)$$

$$= \sum_{m=1}^{\infty} \sum_{n=m}^{\infty} \gamma(\xi_m, k_m) p(\xi_n)\beta(\xi_n, k_n)$$

$$= \sum_{m=1}^{\infty} \gamma(\xi_m, k_m)(1 - P[m - 1, \xi])$$

since  $P[0, \xi] = 0$ . If  $\gamma(j, k) = 1$  then this reduces to

$$\mu(\xi) = \sum_{n=0}^{\infty} (1 - P[n, \xi]).$$

Now I can exhibit the solution explicitly. Under the assumptions outlined above if  $q_j$  is the probability of detecting the asteroid after a single examination of field of view number  $j$  (given that it is in field of view number  $j$ ) then, as each inspection is an independent event, the incremental conditional probability of detection  $\beta(j, k) = b(j, k) - b(j, k - 1)$  is given by

$$\beta(j, k) = q_j (1 - q_j)^{k-1} \quad \text{for } j \in J, k = 1, 2, \dots$$

Normalize such that  $\gamma(j, k) = 1 \forall j \in J, k = 1, 2, \dots$ , and suppose that an allocation  $f(j)$  has total cost (i.e., number of inspections)  $K$ ,

$$\sum_{j \in J} f(j) = K.$$

The total probability of detection for this allocation of effort will be

$$\begin{aligned} P[f] &= \sum_{j \in J} p(j) b(j, f(j)) \\ &= \sum_{j \in J} p(j) [1 - (1 - q_j)^{f(j)}]. \end{aligned}$$

Consider the search plan defined by one makes the  $n$ th inspection in field of view number  $i \in J$  such that

$$\begin{aligned} p(i) q_i (1 - q_i)^{r(i, n, \xi)} \\ = \max_{j \in J} p(j) q_j (1 - q_j)^{r(j, n-1, \xi)}. \end{aligned}$$

Then  $\xi = \xi^*$  and is optimal (in all senses). This result is due to Chew (1967). Since  $J$  is finite the existence of an  $i$  satisfying the above is guaranteed. If one exploits the uniformity of the target distribution  $p$  over the search space  $J$ , then the result is even simpler,

$$\begin{aligned} q_i (1 - q_i)^{r(i, n, \xi)} \\ = \max_{j \in J} q_j (1 - q_j)^{r(j, n-1, \xi)}. \quad (1) \end{aligned}$$

## 5. SEARCH PLAN CONSTRUCTION

I have already argued that the a priori target distribution  $p(j)$  can be approximated by a defective uniform distribution over the search space. In fact,  $\sum_{j \in J} p(j) = \text{area}(J)/4\pi$ . I have also argued that the incremental cost function is homogeneous over  $J$  and independent of the number of looks,  $\gamma(j, k) = 1$  (in appropriate units). The probability of detection of the minor planet in field of view number  $j$  (given that the asteroid is there) is  $q_j$ . This depends principally on the apparent magnitude of the minor planet and the night sky background. Three effects tend to make asteroids fainter; atmospheric extinction, loss of brightness due to increasing phase angle, and increasing distance (heliocentric or geocentric).

The extinction is modeled as usual,

$$\varepsilon = \varepsilon_z \sec z,$$

where  $z$  is the topocentric zenith distance and  $\varepsilon_z$  is the extinction per unit air mass. I have used a value of 0.13 mag/air mass for  $\varepsilon_z$ . For the phase function in magnitudes I've used the Gehrels and Tedesco (1979) results,

$$\begin{aligned} B(1, 0) &= B(1, \theta) + 0.538 \\ &\quad - 0.134 |\theta|^{0.714} - 7l, \text{ for } |\theta| < 7^\circ \\ B(1, 0) &= B(1, \theta) - \theta l \text{ for } |\theta| \geq 7^\circ, \end{aligned}$$

where  $B(1, 0)$  is the absolute  $B$  magnitude and  $B(1, \theta)$  is the apparent magnitude corrected for phase angle  $\theta$ . The parameter of the linear part of the phase function in magnitudes  $l = 0.039$  mag/deg. The illustrative search plans in Figs. 2 and 3 presume a geocentric distance of 0.5 AU at opposition.

For asteroids very much brighter than our limiting magnitude ( $m - m_L < -1^m$ ) the probability of detection is essentially unity. For asteroids very much fainter than our limiting magnitude ( $m - m_L > 1^m$ ) the probability of detection is essentially zero; see Fig. 1. Hence, the most interesting range from the point of view of planning a search is the regime  $|m - m_L| < 1/2^m$ . The search plans shown in Fig. 2 are for midnight on a winter solstice night and a  $B(1, 0) = m_L - 1^m$

**a NORTH**

145	136	121	100	81	72	60	53	54	59	71	82	99	122	135
139	126	104	80	63	46	38	29	30	37	45	64	79	103	125
137	118	96	70	48	28	20	13	14	19	27	47	69	95	117
131	112	86	62	40	22	10	3	4	9	21	39	61	85	111
132	110	84	58	36	18	8	1	2	7	17	35	57	83	109
138	116	90	66	43	26	12	X 5	6	11	25	44	65	89	115
140	124	102	76	50	34	24	15	16	23	33	49	75	101	123
146	134	114	92	74	52	42	31	32	41	51	73	91	113	133
149	144	128	108	94	77	68	55	56	67	78	93	107	127	143
150	148	142	130	120	105	98	87	88	97	106	119	129	141	147

**SOUTH**

**WEST** **EAST**

**RIGHT ASCENSION**

**b NORTH**

145	136	121	100	81	72	60	53	54	59	71	82	99	122	135
139	126	104	80	63	46	38	29	30	37	45	64	79	103	125
137	118	96	70	48	27	20	13	14	19	28	47	69	95	117
131	112	86	62	40	21	10	3	4	9	22	39	61	85	111
132	110	84	58	35	17	8	1	2	7	18	36	57	83	109
138	116	90	66	43	25	12	X 5	6	11	26	44	65	89	115
140	124	102	76	50	34	24	15	16	23	33	49	75	101	123
146	134	114	92	74	52	42	31	32	41	51	73	91	113	133
149	144	127	108	94	77	68	55	56	67	78	93	107	128	143
202	148	142	130	120	105	98	87	88	97	106	119	129	141	147

**SOUTH**

**WEST** **EAST**

**RIGHT ASCENSION**

FIG. 2. Search plans for a bright (a) and a fainter, by 0<sup>m</sup>6, (b) asteroid at midnight on a winter solstice night. The number(s) in the boxes are those examinations of the optimal search plan this field of view of the search space examined.

(Fig. 2a), or  $m_L - 1/2^m$  (Fig. 2b). The search space  $J$  was chosen to be the  $20^\circ \times 2^h$  (declination  $\times$  right ascension) area on the celestial sphere centered at opposition. Note that this prejudices the search plan toward the intuitively obvious region of the celestial sphere. (The latitude of our observatory is  $33^\circ 49'$ .) Each field of view of the search

space is a square,  $2^\circ$  per side, and there are 150 fields of view in the search space. We look first in the field of view with the highest probability of detection and choose subsequent fields of view based on Eq. (1). A simple, repetitive enumeration through Eq. (1) determines the field of view order. Figure 3 shows the  $m_L - 1/2^m$  case at midnight

DECLINATION										RIGHT ASCENSION										EAST																																																								
NORTH										WEST										EAST																																																								
108	98	84	66	54	46	36	30	29	35	45	53	65	83	97	230	215	197	168	145	125	116	127	126	117	144	167	196	214	319	297	265	298	330	351	366	385	384	367	352	331	299	264	296	409	427	460	500	553							554	501	461	428	565																	
107	92	78	62	48	34	24	18	17	23	33	47	61	77	91	229	209	182	159	131	120	142	154	155	143	130	156	181	206	316	285	272	310	345	378	402	454	455	403	346	311	273	238	268	410	438	471	507								508	508	472	434	404	586																
109	90	74	58	40	26	14	11	12	13	25	39	57	73	89	231	205	178	149	113	138	169	184	183	170	112	148	177	204	320	283	278	323	362	396	496	551	550	497	363	324	279	262	204	404	442	483	533								534	534	484	443	404	562																
164	94	76	56	38	20	10	3	4	9	19	37	55	75	93	240	211	180	147	111	152	185	219	218	186	153	110	146	179	210	334	289	276	327	364	449	567			568	385	328	277	265	206	415	436	477	542								543	543	478	437	405	537															
183	102	80	60	44	22	6	2	1	5	21	43	59	79	101	247	223	188	157	119	150	194	236	237	195	118	156	187	222	342	303	270	312	358	425	578			579	359	313	271	262	202	420	418	466	509								510	510	467	419	402	529	572															
224	133	88	72	50	32	16	X	8	15	31	49	71	87	132	259	233	201	176	137	122	162	192	191	163	136	175	200	232	357	322	281	286	339	380	473	577	576	474	381	340	287	280	221	448	406	446	490	569							570	491	447	405	351	518	548															
248	207	106	86	68	52	42	28	27	41	51	67	85	105	206	329	258	228	199	172	141	115	128	129	114	171	198	227	253	489	350	315	275	294	337	360	387	388	361	274	314	349	379	321	518	548	559	490	569	561						495	452	412	433	349	583	532															
341	252	217	135	96	82	70	64	63	69	81	95	134	216	261	435	369	356	269	250	239	226	220	221	225	248	243	260	368	571	536	519	546	524	540	563	575	574	564	541	523	527	466	412	604	571	536	519	546	524	540	563	574	564	541	523	527	466	412	349	655	535															
435	369	356	269	250	239	226	220	221	225	248	243	260	368	571	536	519	546	524	540	563	575	574	564	541	523	527	466	604	571	536	519	546	524	540	563	574	564	541	523	527	466	412	655	535																																
485	389	385	269	250	239	226	220	221	225	248	243	260	368	580	511	480	395	348	333	309	300	301	308	332	347	376	398	655	535																																															
580	513	487	459	430	414	416	422	421	417	413	429	458	486	512	655	535														700	650	615	570	525	480	435	390	345	300	255	210	165	120	75	800	750	705	660	615	570	525	480	435	390	345	300	255	210	165	120																
583	503	445	371	316	258	246	242	241	245	257	317	370	444	502	800	750	705	660	615	570	525	480	435	390	345	300	255	210	850	800	755	710	665	620	575	530	485	440	395	350	305	260	215	170	900	850	805	760	715	670	625	580	535	490	445	400	355	310	265	220																
539	539	489	401	372	344	307	263	262	306	343	373	400	488	538	900	850	805	760	715	670	625	580	535	490	445	400	355	310	950	900	855	810	765	720	675	630	585	540	495	450	405	360	315	270	1000	950	905	860	815	770	725	680	635	590	545	500	455	410	365	320																

Fig. 3. Same format as Fig. 2 except at midnight on a summer solstice night for  $m = m_1 - 1/2^m$ .

on a summer solstice night. All of these examples presented as a concrete elucidation of the formulism were arbitrarily terminated as soon as each field of view of the search space had been examined (for clarity in presenting the plans diagrammatically). Optimal search plans of this nature never terminate but optimal searches do (when the sought for object is found!).

A logical question to ask at this point is "How inefficient is the unplanned search relative to the optimal search?" For our searches, we would've started at opposition and then spiralled outward until each field of view had been examined once. When the asteroid is relatively bright (Fig. 2) this is roughly the same as the optimal search plan. The search plan exhibited in Fig. 2a has a cumulative probability of detection of 92.4%. For the search plan shown in Fig. 2b the cumulative probability of detection is lower, 85.3%, and the usual search plan is 5.5% less efficient still. The comparison of the two search plans for the case of Fig. 3 is more complicated because we would have never repeated an examination of a field of view. With comparable effort to the optimal search plan, but randomly distributed over the search space, the optimal plan starts out more efficient and then becomes comparable to the repeated uniform in areal coverage one. This is typical of extended optimal search plans for medium-bright objects. When one is trying to fully reach one's limits, optimal search plans invest tremendous allocations of effort repeatedly near opposition (since each examination of a field of view is an independent event and the sought for asteroid is at the limits of detection). The plans tend to be factors of 2-4 times as efficient of the uniform ones.

## 6. GENERALIZATIONS

It is clear that any search for a fixed object, from variable stars to geosynchronous artificial satellites, can be cast into this formalism. It should be just as clear that this paper contains all of the essential mathe-

tics for simple searches of this type. Searches for moving objects and multiple observatory searches for the same moving objects can also be solved by similar methods. They are, however, much more difficult to formulate and specify especially since their theoretical structure is incomplete. A more relevant problem is the multiple night search (by the same observatory) for a fixed object. One can plan such searches by an iterative algorithm that takes into account the (presumed) failure of the search. The posterior target distribution, given failure to detect, is updated by Bayes's formula and the conditional detection probability is appropriately modified too. In this fashion a whole week's worth of searching can be optimized. These techniques are applicable to all types of searches (x-ray bursters to comets), and can handle false targets, approximations to optimal plans by incremental means, etc.

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