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Optimal Searches for Asteroids

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Optimal searches for a fixed object are discussed and the rigorous analytical results of discrete search theory are presented. They show that the totally optimal, the uniformly optimal, the locally optimal, and the fastest searches are identical under not too restrictive assumptions. The mathematical formalism is illustrated by an Earth-approaching asteroid search and optimal searches for such objects are explicitly constructed. The approximation that Earth-approaching asteroids are fixed is equivalent to having a very high ($\gtrsim$100 square degrees/hr) search rate. Generalizations to other types of astronomical search are briefly mentioned.

1. CONTEXT

Searches for asteroids, especially Earth-approaching asteroids, are routinely carried out by an MIT group[1] (Taff and Sorvari, 1980; Taff 1980a, b, 1981; various Minor Planet Circulars), Helin and Shoemaker (1979), and others. Although techniques differ (video signal processing for us versus the traditional procedures utilizing photographic plates for other groups), limiting magnitudes differ, and search rates differ, all groups have constrained their searches. In particular we all tend to look near the opposition point, during the new moon phase, and especially in the winter months. These limitations increase the brightness of the sought after minor planet and decrease that of the night sky background both by limiting scattered light and by minimizing background sources of light. The questions addressed in this paper are "Are these searches optimal? *Is* there an optimal search? In what sense is it optimal? How can it be executed?" The answers are "No. Yes. Several (and they lead to the *same* search plan). Simply."

The branch of mathematics that deals with search theory is operations research and I shall assume that the reader is not well versed in such matters. Hence, a large part of this paper is, necessarily, an introduction to the theory of search. There exists an excellent reference on the subject by L. D. Stone (1975). In order to ease the transition for the reader from Stone's book to this paper I have followed his notation. Proofs and supplementary material can be found therein.

Below I have formulated what is known as the search with discrete effort for a fixed target. Asteroid searches are used as a model to illustrate the mathematical formalism. Relatively little simplification of the physics or astronomy is necessary to do this. Next the results of optimal search theory are stated and the optimal search problem is solved. Following this optimal searches for Earth-approaching asteroids are *explicitly* constructed and exhibited. Lastly, generalizations of optimal searches in this and in other astronomical contexts are briefly considered.

2. FORMULATION

One looks for asteroids on the celestial sphere. In the largest sense this forms the two-dimensional *search space* of the problem. In practice we delineate a limited area of the celestial sphere (say above altitude

[1] Myself, D. E. Beatty, R. L. Irelan, R. C. Ramsey, and J. M. Sorvari.

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30°) that we shall actually search in. Denote this search space by J.

One searches using a telescope with a finite field of view. In practice we always examine an entire field, never a fraction of a field nor more than one field at a time. Hence the search space J consists of a discrete set of fields of view. Number these by the index $j = 1, 2, \ldots$. In particular, since the celestial sphere encompasses 4π sr, max $(j) < \infty$.

Before the minor planet is found one assigns an a priori *target distribution* on the search space J, $p: J \rightarrow [0,1]$ (the notation means that p is a function defined over the set J which maps elements of J into the domain zero to unity inclusive). The target distribution is the a priori probability of finding an asteroid in field of view $j \varepsilon J$ before one starts the search. For main belt asteroids a reasonable model for p is p is uniform over all heliocentric ecliptic longitudes and over the heliocentric ecliptic latitude range <10° (or 5° or 20°). For Earth-approaching minor planets, both because of parallax effects and the inherent spread of Earth-approaching asteroids over orbital element space (particularly in inclination), a reasonable model for P is that P is uniform over the topocentric celestial sphere. In any case

$$\sum_{j \varepsilon J} p(j) \leq 1.$$

When one examines a field of view (whether a video frame or an exposed plate) for an asteroid one expends a certain amount of effort trying to detect the asteroid. In the photographic case one is looking for the streak that the moving minor planet has left. In the video mode one looks for two displaced dots from frames taken at different times (and after the stars have been electronically subtracted). One may look at the same field of view several times. The cost of performing k inspections in the jth field of view is measured by a *cost function* $c(j,k)$: $J \times \{0,1,2, \ldots\} \rightarrow [0,\infty]$. Clearly $c(j,0) = 0 \; \forall j \varepsilon J$ (no effort implies no cost). One could measure cost by the time spent examining a field of view plus the time spent in moving to the next field of view (this makes c nonlocal and is not desirable). Operationally we always spend the same time in each field of view (more or less). Also, because [area $(J)]^{1/2}$/slew speed $\ll$ time spent examining a field of view, the nonlocal element of c is both unimportant and varies little. We typically spend 45 sec examining a field, the telescope slews at 4°/sec, and we rarely search more than 500 square degrees per night. Hence [area $(J)^{1/2}]$/slew speed = 5.6 sec. (For photographic searches it is a good approximation too because large plates are usually used; i.e., 6° × 6°.) Thus I shall measure cost by time and, in the instance of the asteroid search, specialize to the case when the incremental cost of the kth examination in field of view number j, viz.,

$$\gamma(j,k) = c(j,k) - c(j,k-1)$$

is a constant independent of both j (i.e., the telescope is fast or the plates are large and all fields of view are treated equally) and k (e.g., the same field of view is equally well inspected each time).

When one does examine a field of view of the search space looking for an asteroid then there is a conditional probability of detecting it on or before the kth inspection of that field of view (given that it is there). This function, for field of view number j and examination k, is denoted by $b(j,k)$: $J \times \{0,1,2, \ldots\} \rightarrow [0,1]$. Naturally $b(j,0) = 0 \; \forall j \varepsilon J$ (you cannot find it if you do not look for it). From the *detection function b* one can construct the probability of failing to detect the asteroid on the first $k - 1$ scrutinizations of field of view number j and then succeeding on the kth one (given that the asteroid is in field of view number j); viz.,

$$\beta(j,k) = b(j,k) - b(j,k-1).$$

There is a lot of physics and mathematics subsumed in the detection function. Clearly it depends on the asteroid's apparent mag-

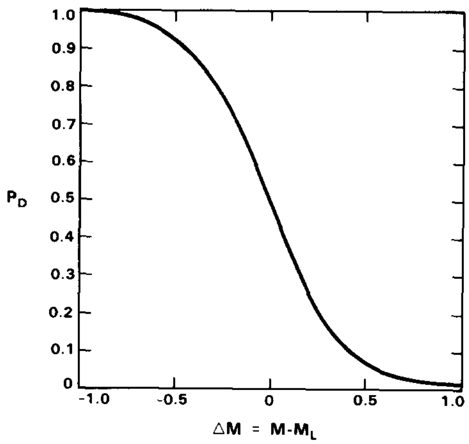

FIG. 1. Probability of detection as a function of "distance" from limiting magnitude m_L. The functional form is constant/$\{1 + \exp[5(m - m_L)]\}$.

nitude, the background star density, the night sky background brightness, the resolution element size of the detector(s), the false alarm probability one is willing to accept, how tired one is, etc. Since the ecliptic is unchanging, atmospheric extinction can be computed, the Moon's position is known, etc., this is a computable function. Indeed we are developing software to realistically do so in a physically correct way. Operationally, for a fixed set of external parameters, our detection probability has the shape shown in Fig. 1 where m_L is our quoted limiting magnitude (e.g., where the probability of detection is 50%). The form shown in the diagram will be used to compute the optimal search plans given below.

Finally I need to define a *search plan.* A discrete search plan is a sequence $\xi = (\xi_1, \xi_2, \xi_3, \ldots)$ which tells the searcher to first look in cell ξ_1; if the asteroid is found there then terminate the search but if the minor planet isn't found then look next in field of view ξ_2, etc. A global way to describe this is by a function which specifies the *allocation* of effort devoted to each field of view j. To this end, define f: $J \rightarrow [0,\infty)$; $f(j)$ is the number of examinations in field of view number j.

Above I referred to searches for a fixed target. Clearly the minor planets we are trying to find are moving. Earth-approaching asteroids can have geocentric angular speeds of a degree per day (or more). I have made the assumption that even these objects are fixed when compared to our search rate. The mathematical formulation of this approximation is [area (J)/search rate] · asteroid angular speed $\ll$ field of view. For our parameters [area (J) = 500 square degrees, search rate = 100 square degrees/hr, asteroid angular speed = 1°day, field of view = 2°] an asteroid could traverse only one-twelfth of a field of view before the entire search is completed. Hence the real problem fits into the formalism reasonably well. For faster moving minor planets the optimal search plans developed below need to be corrected for the asteroid's motion.

3. OPTIMAL SEARCHES

Given the cost of searching field of view number j a total of k times, $c(j,k)$, the total cost of performing the search plan ξ with allocation f is

$$C[f] = \sum_{j\epsilon J} c(j,f(j)).$$

The total number of examinations over all fields of view is $\Sigma_{j\epsilon J} f(j)$. Similarly the total probability of minor planet detection with this allocation of effort is $P[f]$,

$$P[f] = \sum_{j\epsilon J} p(j)b(j,f(j)),$$

where $b(j,k)$ is the conditional probability of finding the asteroid in field of view number j after k examinations of that field of view given that it's in that field of view.

There are four types of searches one might define as optimal. One might be interested in maximizing the total probability of detection when constrained to a given number of inspections (say K). If the incremental cost function $\gamma(j,k) = c(j,k) - c(j,k-1)$ is a constant, then (after a suitable renormalization) one is demanding that $P[f]$ be a maximum for $C[f] \le K$. Such a search is termed *totally optimal.* If one demanded optimality for all $K = 1, 2, 3, \ldots$, then the search is called *uniformly optimal.* A third type of search plan that one might consider is the search plan that maximizes the probability of detection with respect to the incremental cost and does so at evey step of the search. Mathematically one finds the value of j which maximizes $p(j)\beta(j,k)/\gamma(j,k)$ at each k. These searches are called *locally optimal.* Lastly one might entertain a search plan that minimized the total expected cost (i.e., was *the fastest*) to find the target.

The essential assumptions necessary to cast the asteroid search into the simplest form of the mathematical superstructure that Stone (1975) outlines are

(1) that the asteroid is stationary (i.e., search rate is high compared to the asteroid's angular speed),

(2) that the search space is discrete (i.e., a fixed field of view),

(3) that the allocation of effort is discrete (i.e., no favored fields of view), and

(4) that γ is bounded away from zero and $p(j)b(j,k)/\gamma(j,k)$ is a decreasing function of j (i.e., no free examinations of a field of view and the larger the search space the more difficult to detect).

I do not believe that the physics or astronomy is strained by these strictures. In fact (5) γ = constant is not unreasonable (i.e., the telescope moves smartly). The important point is that under these five limitations the totally optimal search plan, the uniformly optimal search plan, the locally optimal search plan, and the fastest searches are all *identical.* Not only that, it can be explicitly exhibited. See Stone's text for the rigorous mathematical statements of the relevant theorems and their proofs.

4. THE SEARCH PLAN

I need just a bit more mathematics before I can exhibit the solution to the optimal search problem. The search plan $\xi = (\xi_1, \xi_2, \xi_3, \ldots)$ is a sequence of values $\xi_i \in J$ for $i = 1,2,3, \ldots$. These specify that the ith examination be in field of view ξ_i if the previous $i - 1$ inspections failed to detect the asteroid in fields of view $\xi_1, \xi_2, \ldots, \xi_{i-1}$. Let the set of all such search plans be denoted by Ξ. Introduce the probability $P[n, \xi]$ (and the cost $C[n, \xi]$) of detecting the asteroid on or before the nth examination while performing search plan $\xi \in \Xi$ (of the first n inspections). Finally, let $r(j,n,\xi)$ be the number of scrutinizations out of the first n that are placed in the jth field of view while following search plan ξ. A uniformly optimal search plan [for $\gamma(j,k) = 1$; this is an unimportant normalization] $\xi^* \in \Xi$ is one such that

$$P[n,\xi^*] = \max\{P[n,\xi] : \xi \in \Xi\}, \qquad n = 1, 2, \ldots, K.$$

A locally optimal search plan ξ^* is one such that ξ_1 is determined by [$\gamma \neq 0$ necessarily]

$$\frac{p(\xi_1)\beta(\xi_1,1)}{\gamma(\xi_1,1)} = \max_{j \in J} \frac{p(j)\beta(j,1)}{\gamma(j,1)}$$

and having determined the field of view for the first $n - 1$ examinations $(\xi_1, \xi_2, \ldots, \xi_{n-1})$ the field of view for the nth one is determined from

$$\frac{p(i)\beta(i,r(i,n-1,\xi)+1)}{\gamma(i,r(i,n-1,\xi)+1)} = \max_{j \in J} \frac{p(j)\beta(j,r(j,n-1,\xi)+1)}{\gamma(j,r(j,n-1,\xi)+1)}$$

with $\xi_n^* = i$. Now define $k_n = r(\xi_n, n, \xi)$. The notation means that the nth examination of the search plan ξ is placed in field of view ξ_n and that it is the k_nth time that this field of view has been searched. The average cost to find the asteroid can be expressed in a variety of ways if the limit as $n \to \infty$ of $P[n,\xi]$ is unity;

$$\begin{aligned}
\mu(\xi) &= \sum_{n=1}^{\infty} C[n,\xi](P[n,\xi] - P[n-1,\xi]) \\
&= \sum_{n=1}^{\infty} \sum_{m=1}^{n} \gamma(\xi_m,k_n)p(\xi_n)\beta(\xi_n,k_n) \\
&= \sum_{m=1}^{\infty} \sum_{n=m}^{\infty} \gamma(\xi_m,k_n)p(\xi_n)\beta(\xi_n,k_n) \\
&= \sum_{m=1}^{\infty} \gamma(\xi_m,k_m)(1 - P[m-1,\xi])
\end{aligned}$$

since $P[0,\xi] = 0$. If $\gamma(j,k) = 1$ then this reduces to

$$\mu(\xi) = \sum_{n=0}^{\infty} (1 - P[n,\xi]).$$

Now I can exhibit the solution explicitly. Under the assumptions outlined above if q_j is the probability of detecting the asteroid after a single examination of field of view number j (given that it is in field of view number j) then, as each inspection is an independent event, the incremental conditional probability of detection $\beta(j,k) = b(j,k) - b(j,k-1)$ is given by

$$\beta(j,k) = q_j\,(1 - q_j)^{k-1} \quad \text{for } j \in J,\ k = 1, 2, \ldots .$$

Normalize such that $\gamma(j,k) = 1\ \forall j \in J$, $k = 1, 2, \ldots$, and suppose that an allocation $f(j)$ has total cost (i.e., number of inspections) K,

$$\sum_{j\in J} f(j) = K.$$

The total probability of detection for this allocation of effort will be

$$P[f] = \sum_{j\in J} p(j)b(j,f(j)) = \sum_{j\in J} p(j)[1 - (1 - q_j)^{f(j)}].$$

Consider the search plan defined by one makes the nth inspection in field of view number $i \in J$ such that

$$p(i)q_i(1 - q_i)^{r(i,n,\xi)} = \max_{j\in J} p(j)q_j(1 - q_j)^{r(j,n-1,\xi)}.$$

Then $\xi = \xi^*$ and is optimal (in all senses). This result is due to Chew (1967). Since J is finite the existence of an i satisfying the above is guaranteed. If one exploits the uniformity of the target distribution p over the search space J, then the result is even simpler,

$$q_i(1 - q_i)^{r(i,n,\xi)} = \max_{j\in J} q_j(1 - q_j)^{r(j,n-1,\xi)}. \quad (1)$$

5. SEARCH PLAN CONSTRUCTION

I have already argued that the a priori target distribution $p(j)$ can be approximated by a defective uniform distribution over the search space. In fact, $\Sigma_{j\in J}\,p(j) = \text{area}\,(J)/4\pi$. I have also argued that the incremental cost function is homogeneous over J and independent of the number of looks, $\gamma(j,k) = 1$ (in appropriate units). The probability of detection of the minor planet in field of view number j (given that the asteroid is there) is q_j. This depends principally on the apparent magnitude of the minor planet and the night sky background. Three effects tend to make asteroids fainter; atmospheric extinction, loss of brightness due to increasing phase angle, and increasing distance (heliocentric or geocentric).

The extinction is modeled as usual,

$$\varepsilon = \varepsilon_z \sec z,$$

where z is the topocentric zenith distance and ε_z is the extinction per unit air mass. I have used a value of 0.13 mag/air mass for ε_z. For the phase function in magnitudes I've used the Gehrels and Tedesco (1979) results,

$$B(1,0) = B(1,\theta) + 0.538 - 0.134\,|\theta|^{0.714} - 7l, \text{ for } |\theta| < 7^\circ$$

$$B(1,0) = B(1,\theta) - \theta l \text{ for } |\theta| \geq 7^\circ,$$

where $B(1,0)$ is the absolute B magnitude and $B(1,\theta)$ is the apparent magnitude corrected for phase angle θ. The parameter of the linear part of the phase function in magnitudes $l = 0.039$ mag/deg. The illustrative search plans in Figs. 2 and 3 presume a geocentric distance of 0.5 AU at opposition.

For asteroids very much brighter than our limiting magnitude ($m - m_L < -1^m$) the probability of detection is essentially unity. For asteroids very much fainter than our limiting magnitude ($m - m_L > 1^m$) the probability of detection is essentially zero; see Fig. 1. Hence, the most interesting range from the point of view of planning a search is the regime $|m - m_L| < 1/2^m$. The search plans shown in Fig. 2 are for midnight on a winter solstice night and a $B(1,0) = m_L - 1^m$

a NORTH

DECLINATION

145	136	121	100	81	72	60	53	54	59	71	82	99	122	135
139	126	104	80	63	46	38	29	30	37	45	64	79	103	125
137	118	96	70	48	28	20	13	14	19	27	47	69	95	117
131	112	86	62	40	22	10	3	4	9	21	39	61	85	111
132	110	84	58	36	18	8	1	2	7	17	35	57	83	109
138	116	90	66	43	26	12	5	6	11	25	44	65	89	115
140	124	102	76	50	34	24	15	16	23	33	49	75	101	123
146	134	114	92	74	52	42	31	32	41	51	73	91	113	133
149	144	128	108	94	77	68	55	56	67	78	93	107	127	143
150	148	142	130	120	105	98	87	88	97	106	119	129	141	147

SOUTH

WEST EAST

RIGHT ASCENSION

b NORTH

DECLINATION

145	136	121	100 187	81 154	72 166	60 194	53	54	59 195	71 167	82 155	99 186	122	135
139	126	104 193	80 153	63 185	46	38	29	30	37	45	64 184	79 152	103 192	125
137	118	96 177	70 170	48	27	20	13	14	19	28	47	69 171	95 176	117
131	112	86 161	62 190	40	21	10	3	4	9	22	39	61 191	85 160	111
132	110	84 158	58 198	35	17	8	1	2	7	18	36	57 199	83 159	109
138	116	90 169	66 182	43	25	12	5	6	11	26	44	65 183	89 168	115
140	124	102 188	76 156	50	34	24	15	16	23	33	49	75 157	101 189	123
146	134	114	92 173	74 164	52	42	31	32	41	51	73 165	91 172	113	133
149	144	127	108	94 175	77 151	68 180	55 201	56 200	67 181	78 150	93 174	107	128	143
202	148	142	130	120	105 196	98 179	87 162	88 163	97 178	106 197	119	129	141	147

SOUTH

WEST EAST

RIGHT ASCENSION

FIG. 2. Seach plans for a bright (a) and a fainter, by $0^{m}\!.6$, (b) asteroid at midnight on a winter solstice night. The number(s) in the boxes are those examinations of the optimal search plan this field of view of the search space examined.

(Fig. 2a), or $m_L - 1/2^m$ (Fig. 2b). The search space J was chosen to be the $20° \times 2^h$ (declination × right ascension) area on the celestial sphere centered at opposition. Note that this prejudices the search plan toward the intuitively obvious region of the celestial sphere. (The latitude of our observatory is 33°49′.) Each field of view of the search space is a square, 2° per side, and there are 150 fields of view in the search space. We look first in the field of view with the highest probability of detection and choose subsequent fields of view based on Eq. (1). A simple, repetitive enumeration through Eq. (1) determines the field of view order. Figure 3 shows the $m_L - 1/2^m$ case at midnight

NORTH

DECLINATION

108 230 319 409 555	98 215 297 427	84 197 265 460	66 168 298 500	54 145 330 553	46 125 351	36 116 366	30 127 385	29 126 384	35 117 367	45 124 352	53 144 331 554	65 167 299 501	83 196 264 461	97 214 296 428
107 229 318 410 556	92 209 285 438	78 182 272 471	62 159 310 507	48 131 345	34 120 378	24 142 402	18 154 454	17 155 455	23 143 403	33 121 379	47 130 346	61 158 311 508	77 181 273 472	91 208 284 439
109 231 320 404 552	90 205 283 442	74 178 278 483	58 149 323 533	40 113 362	26 138 396	14 169 496	11 184 551	12 183 550	13 170 497	25 139 397	39 112 363	57 148 324 534	73 177 279 484	89 204 282 443
164 240 334 415 537	94 211 289 436	76 180 276 477	56 147 327 542	38 111 364	20 152 449	10 185 567	3 219	4 218	9 186 568	19 153 450	37 110 365	55 146 328 543	75 179 277 478	93 210 288 437
193 247 342 420 529	102 223 303 418 572	80 188 270 466	60 157 312 509	44 119 358	22 150 425	6 194 578	2 236	1 237	5 195 579	21 151 426	43 118 359	59 156 313 510	79 187 271 467	101 222 302 419 573
224 259 357 448 518	133 233 322 406 548	88 201 281 446	72 176 286 490	50 137 339 569	32 122 380	16 162 473	7 192 577	8 191 576	15 163 474	31 123 381	49 136 340 570	71 175 287 491	87 200 280 447	132 232 321 405 549
248 329 389 468 532	207 254 350 434 521	106 228 315 411 559	86 199 275 451	68 172 294 494	52 141 337 561	42 115 360	28 128 387	27 129 388	41 114 361	51 140 338 562	67 171 295 495	85 198 274 452	105 227 314 412 560	206 253 349 433 522
341 386 453 504 571	252 336 391 470 536	217 256 354 441 519	135 235 326 408 546	96 213 291 431	82 190 267 462	70 174 292 492	64 166 304 505	63 165 305 506	69 173 293 493	81 189 266 463	95 212 290 432	134 234 325 547	216 255 353 440 520	251 335 390 469 535
435 485 511 580	369 399 480 513	261 356 395 487 558	244 269 377 459 528	203 250 348 430 524	161 239 333 414 540	104 226 309 416 563	99 220 300 422 575	100 221 301 421 574	103 225 308 417 564	160 238 332 413 541	202 249 347 429 523	243 268 376 458 527	260 355 394 486 557	368 398 479 512
583	503 539	445 489 517 582	371 401 482 515	316 372 423 498 565	258 344 393 476 545	246 307 383 465 531	242 263 375 457 526	241 262 374 456 525	245 306 382 464 530	257 343 392 475 544	317 373 424 499 566	370 400 481 514	444 488 516 581	502 538

SOUTH

WEST | RIGHT ASCENSION | EAST

FIG. 3. Same format as Fig. 2 except at midnight on a summer solstice night for $m = m_L - 1/2^m$.

on a summer solstice night. All of these examples presented as a concrete elucidation of the formulism were arbitrarily terminated as soon as each field of view of the search space had been examined (for clarity in presenting the plans diagrammatically). Optimal search plans of this nature never terminate but optimal searches do (when the sought for object is found!).

A logical question to ask at this point is "How inefficient is the unplanned search relative to the optimal search?" For our searches, we would've started at opposition and then spiralled outward until each field of view had been examined once. When the asteroid is relatively bright (Fig. 2) this is roughly the same as the optimal search plan. The search plan exhibited in Fig. 2a has a cumulative probability of detection of 92.4%. For the search plan shown in Fig. 2b the cumulative probability of detection is lower, 85.3%, and the usual search plan is 5.5% less efficient still. The comparison of the two search plans for the case of Fig. 3 is more complicated because we would have never repeated an examination of a field of view. With comparable effort to the optimal search plan, but randomly distributed over the search space, the optimal plan starts out more efficient and then becomes comparable to the repeated uniform in areal coverage one. This is typical of extended optimal search plans for medium-bright objects. When one is trying to fully reach one's limits, optimal search plans invest tremendous allocations of effort repeatedly near opposition (since each examination of a field of view is an independent event and the sought for asteroid is at the limits of detection). The plans tend to be factors of 2–4 times as efficient of the uniform ones.

6. GENERALIZATIONS

It is clear that any search for a fixed object, from variable stars to geosynchronous artificial satellites, can be cast into this formalism. It should be just as clear that this paper contains all of the essential mathematics for simple searches of this type. Searches for moving objects and multiple observatory searches for the same moving objects can also be solved by similar methods. They are, however, much more difficult to formulate and specify especially since their theoretical structure is incomplete. A more relevant problem is the multiple night search (by the same observatory) for a fixed object. One can plan such searches by an iterative algorithm that takes into account the (presumed) failure of the search. The posterior target distribution, given failure to detect, is updated by Bayes's formula and the conditional detection probability is appropriately modified too. In this fashion a whole week's worth of searching can be optimized. These techniques are applicable to all types of searches (x-ray bursters to comets), and can handle false targets, approximations to optimal plans by incremental means, etc.

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