

© 1980 IEEE. Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution to servers or lists or to reuse any copyrighted component of this work in other works must be obtained from the IEEE.

This material is presented to ensure timely dissemination of scholarly and technical work. Copyright and all rights therein are retained by authors or by other copyright holders. All persons copying this information are expected to adhere to the terms and constraints invoked by each author's copyright. In most cases, these works may not be reposted without the explicit permission of the copyright holder.

COMPARISON OF VARIOUS ELEVATION ANGLE  
ESTIMATION TECHNIQUES

J.E. Evans  
D.F. Sun

M.I.T. Lincoln Laboratory

ABSTRACT

The angular resolution and tracking of closely spaced targets is a classical radar problem which is receiving increased attention; and terrain multipath (e.g., reflections) has long been recognized to be a principal limitation on the achievable accuracy of radar elevation trackers at low elevation angles. A variety of techniques have been proposed for improved elevation angle estimation; however, comparative analysis based on field comparable data has not been available to date. In this paper, distributions of multipath scattered power, described in a companion paper, are used to compare several elevation angle estimation techniques:

- (1) conventional monopulse;
- (2) off-boresight monopulse;
- (3) double null monopulse;
- (4) single edge processing as is used for flare processing in the Microwave Landing System; and
- (5) a maximum entropy technique based estimator.

I. INTRODUCTION

Low angle elevation trackers have been under active investigation for many years [1]. New applications for low angle tracking in the civilian (e.g., MLS flare [2]) and military (e.g., protection against cruise missiles or low flying aircraft) sectors have increased interest in improved performance. Additionally, the decreasing costs and increasing speed of digital signal processing has made many approaches practically feasible which hitherto would primarily have been only of theoretical interest. Consequently, a number of new techniques have been proposed as an alternative to the conventional monopulse or conical scan.

Unfortunately, most studies of new angle estimation techniques have not been comparative in

This work was sponsored by the Federal Aviation Administration. "The views and conclusions contained in this document are those of the contractor and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the United States Government."

nature; and, the bulk of the comparative studies to date have consisted of simulating performance against a single specular reflection from a flat homogeneous surface. By contrast, natural terrain multipath environments are characterized by a variety of multipath sources whose distribution of scattered power as a function of elevation angle can be quite different from that for a flat surface [3]. In those cases where comparative field tests have been made, the comparative analysis was often complicated by differences in target flight profile and reference system\* as well as changes in the multipath environment between the respective tests [4].

Our approach to the comparative analysis has been to measure the received waveforms at various points along the aperture of a line array and then realize various angle estimators in software. Since all the various estimates are derived from the same field data, comparative performance differences due to flight profile, environment conditions and reference system errors are thus eliminated. Additionally, if a given technique is further optimized, one can assess the performance improvements without repeating the field tests. Similarly, tradeoff studies of the performance versus aperture size can be carried out on an identical base.

Section II describes the measurement system and procedures used to obtain the aperture sampled field data. Section III describes the angle estimation techniques considered in the present study and presents and discusses the comparative performance using field data at L and C bands from measurements over vegetated flat and rolling terrain. Section V summarizes the results of our studies to date.

II. MEASUREMENT EQUIPMENT AND EXPERIMENTAL  
PROCEDURE

Fig. 1 shows the aperture sampling equipment utilized in the field measurements. Each of sampled apertures consists of two uniformly spaced interleaved line arrays with the following parameters:

\*e.g., tracking theodolite errors.

Array	Frequency	Signals	Array 1	
	(GHz)		N	$\delta$
L band	1.09	0.45 $\mu$ sec pulse	9	3.24
C band	5.2	CW	19	3.19

Array 2		Array for Comparative Studies		
N	$\delta$	N	$\delta$	BW
5	1.62	5	1.62*	7.09°
21	1.6	10	1.6	3.6°

N = no. of aperture samples  $\delta$  = element spacing in wavelengths  
 BW = standard beamwidth (=  $1/N\delta$ )

The phase\* and log amplitude at each element are digitized (0.7dB granularity in amplitude and 2° in phase) and stored on magnetic tape. The helicopter range is determined from the L band signal while the elevation angle is measured by digitizing the manually controlled optical theodolite position (granularity  $\approx 0.04^\circ$ ). Look-up tables for log amplifiers and phase detectors are obtained before and following each measurement sequence by injecting signals at IF with various attenuations and phase shifts. Antenna collimation is maintained by injecting rf calibration signals at each element of the array and by using a rf design which minimizes changes in electrical length due to temperature.

The helicopter flight profiles typically consisted of vertical ascents/descents along a given radial at distances of 0.5 to 1.0 nmi from the ground antenna. This permitted the terrain along the radial to be measured and modeled in detail for correlation with the MLS propagation model. By using relatively close ranges, a high SNR was obtained (typically 30 dB). Measurements at longer ranges (e.g., 2 nmi) showed minimal variation in the received signal characteristics at a given elevation angle.

### III. ANGLE ESTIMATION TECHNIQUES

Several of the angle estimation techniques can be motivated by reference to the angular power spectra discussed in the companion paper [3]:

- the maximum entropy (ME) technique based estimator takes the largest peak in the ME power spectrum at positive elevation angles to be the angle of the direct signal. It has been shown by Lang [5] that this estimator closely approaches the performance of the true maximum likelihood estimator when the received signal consists of two plane waves in noise.
- conventional null seeking monopulse (CM) systems can be viewed as determining the peak of the beamsum (BS) power

\* relative to the bottom element of the array

spectrum  $|\Sigma(\theta)|^2$  since the null of the ratio  $\Delta(\theta)/\Sigma(\theta)$  with difference pattern  $\Delta(\theta) = d\Sigma(\theta)/d\theta$  occurs at the peak of the beam sum pattern  $\Sigma(\theta)$ .

Since the beamsum spectrum peak corresponding to the direct signal ( $PK_d$ ) is typically biased by multipath on the side of the peak towards the multipath (i.e., at angles less than that of the direct signal), estimators which primarily rely on the less distorted portion of  $PK_d$  (i.e., that at angles above the direct signal) seem sensible. Two such estimators are considered here:

- the MLS "single edge" flare processor (SEP) which uses a delay and compare technique to locate a given slope of the  $PK_d$  [6]. The point used typically is 6dB to 9dB below the nominal peak of  $\Sigma(\theta)$ .
- the off-boresight monopulse (OBM) technique [7] which utilizes the fact that when only a direct signal is present at angle  $\theta_d$  and  $\theta$  is within 1 beamwidth of  $\theta_d$

$$\epsilon(\theta) \triangleq \Delta(\theta)/\Sigma(\theta) \approx (N\delta)(\theta - \theta_d) \quad (1)$$

so that one can estimate  $\theta_d$  without pointing the array at  $\theta_d$ . With an "off-boresight" elevation tracker  $\theta$  is constrained to be  $> 0.7/N\delta$  and (1) is used to estimate  $\theta_d$  if the last estimate of  $\theta_d$  is less than  $0.7/N\delta$ . This keeps the main lobes of  $\Sigma(\theta)$  and  $\Delta(\theta)$  pointed above the terrain and thus significantly reduces the errors due to multipath signals at elevation angles below  $\theta_d$  [1].

At low elevation angles, it may not be possible to prevent the mainlobes of  $\Delta(\theta)$  from illuminating the ground. W. White [8] has suggested use of a double null monopulse (DNM) in which the difference pattern has a null both at  $\theta$  and at the expected angle of the multipath ( $=-\theta$ ). The error metric  $\epsilon(\theta)$  for White's monopulse estimator has a null at  $\theta=0$  when only the direct signal is present and when the multipath is present at  $-\theta_d$ . Good results have been reported for this estimator over sea and desert terrain [8].

Our software implementation of these various techniques closely parallels the various references cited above. The  $\Sigma(\theta)$  used for the CM, SEP, and OBM techniques was a Dolph-Chebyshev (DC) design with -20dB (-30dB for SEP) sidelobes. The  $\Delta(\theta)$  used for CM and OBM was the derivative of a -40dB sidelobe DC design (this had a peak sidelobe of -22dB). The  $\Sigma(\theta)$  and  $\Delta(\theta)$  patterns used for DNM were according to the prescription in [8]. It was found that the residual array and quantization errors increased the small array  $\Sigma(\theta)$  sidelobes enough such that the SEP threshold would have to correspond to -6dB point on the nominal  $\Sigma(\theta)$  pattern.

Fig. 2 shows the simulation results for the various angle estimation techniques for a single multipath signal of relative amplitude  $\rho=0.9$  at angle  $\theta_m$  when the direct signal is at  $\theta_d = 0.5BW$ , with 30dB element SNR. Both 0° and 180° relative

phase conditions are shown since these two bound the errors at other relative phases. The ME technique appears to give the best performance over the range of angles with the SEP and DNM techniques a close second.

#### IV. MEASURED RESULTS

Here we will show some L-band and C-band field measurement results. As mentioned in Section II, the C-band results were obtained with a 10-element  $14.3\lambda$  array while the L-band results were obtained with a 5-element  $6.5\lambda$  array\*. The angular error shown in the following figures is the difference between estimated angle and the theodolite tracking angle at that particular moment. Therefore, it is understood that the angular error also includes the possible theodolite tracking error. The theodolite tracking error is expected to be on the order of  $0.1^\circ$  for the measurements described here.

Fig. 3 shows the errors for the various techniques as a function of elevation angle for a test at L.G. Hanscom Airport, Mass. The terrain here is nominally flat with grass cover. We see that in general all techniques except the SEP have similar performance for  $\theta > 1$  beamwidth (BW), where  $\theta_d$  is the target helicopter elevation angle. The maximum angular error ( $\Delta\theta$  max) is about 0.07 BW, except the SEP which shows larger error of 0.12 BW. For  $1 \text{ BW} > \theta > 0.5 \text{ BW}$ , it appears that the ME, DNM and OBM techniques yield smaller error ( $\Delta\theta$  max  $\approx 0.08$  BW), with the ME technique having the best performance around  $\theta \approx 0.5 \text{ BW}$ . For  $\theta < 0.5 \text{ BW}$ , the ME technique appears to give the best performance with the angular error similar to that observed in the higher elevation angles. The larger angular error in the L-band results (Fig. 3b), especially with the ME technique, probably is due to the insufficient sensor samples (only 5 signal samples available for the L-band versus 10 for the C-band).

Fig. 4 shows the errors for the various techniques as a function of elevation angle for a test at the Ft. Devens, Mass. golf course. This terrain is rolling with closely cropped grass, as to yield specular reflections at several elevation angles. In general, the angular errors are larger for the rolling terrain here than those previously observed for the near-flat terrain. Also, the small number of sensor samples has a more pronounced effect on the L-band angle estimation accuracy here, especially with the ME technique, than previously observed for the near-flat terrain. This is thought reasonable, since the multipath environment was found to be more complicated for this rolling terrain than for the near-flat terrain [3]. For the C-band result (Fig. 4a)

\*The small number of L-band array elements necessitated using a two-pole model for the ME technique, whereas the observed angular spectra, in some cases, would require at least a four-pole model. Consequently, the ME L-band errors are greater than would have been the case with a greater number of elements within the overall  $6.5\lambda$  aperture.

which was obtained with a larger number of sensor samples, we can see that again the ME technique appears to yield the best performance, especially for  $\theta < 1 \text{ BW}$ . The ME angular errors ( $\Delta\theta$  max) are around 0.07 to 0.1 BW, except for one isolated elevation angle ( $\theta \approx 4.0^\circ$ ) where all techniques show a large angular error ( $\approx 0.15 \text{ BW}$ ). Again the SEP gives much larger angular error than the other techniques, as we previously observed. This greater error for SEP is believed to arise from the SEP sensitivity to sidelobe multipath.

#### V. SUMMARY

Our preliminary results from applying five different elevation angle estimation techniques to several identical data sets, both synthetic and field measured, indicated that the ME technique based estimator seemed to yield the best performance if a sufficient number of sensor samples was available. The observed maximum angular errors were around 0.07 to 0.1 beamwidths for the target elevation angles from 2 beamwidths down to about 0.3 beamwidths. The DNM and SEP appeared to work much better than the CM or OBM for the synthetic data cases. However, in the field measurement results, the DNM and OBM seemed to give similar performance and the SEP performed notably poorer than the other optimized elevation angle estimation techniques.

#### REFERENCES

1. D. Barton, Radars, vol. 4: Radar Resolution and Multipath Effects, Artech Press, Dedham, Mass. (1975).
2. R. Kelly and E. LeBerge, "MLS flare low elevation angle guidance considerations", this conference.
3. J. Evans and D. Sun, "Experimental measurement of the low angle terrain scattering interference environment", this conference.
4. "DMLS/TRSB comparative demonstration test results at Kjevik Airport, Kristiansand, Norway", ICAO All Weather Operations Divisional Meeting Working Paper 136 (1978).
5. S. Lang, "Near optimum frequency/angle of arrival estimates based on maximum entropy spectral techniques" ICASSP 80 Proceedings, pp. 829-832 (1980).
6. J. Beneke, D. Wightman, A. Offt and C. Vallone, "TRSB multimode digital processor" FAA-RD-77 (April, 1978).
7. P. Dax, "Keep track of that low-flying attack", Microwaves, pp. 36-53 (1976).
8. W.D. White, "Low Angle Radar Tracking in the Presence of Multipath", IEEE Trans on AES., Vol. AES-10, P. 835, (Nov. 1974).

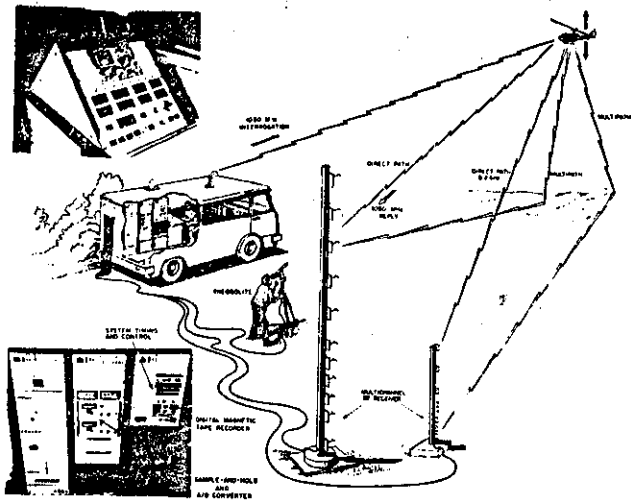


Fig. 1. Aperture sampling experimental configuration.

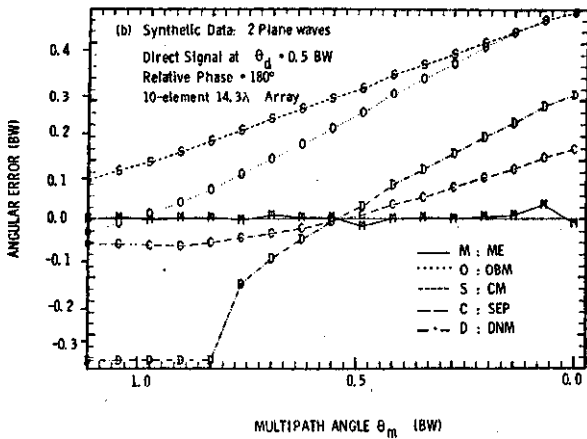
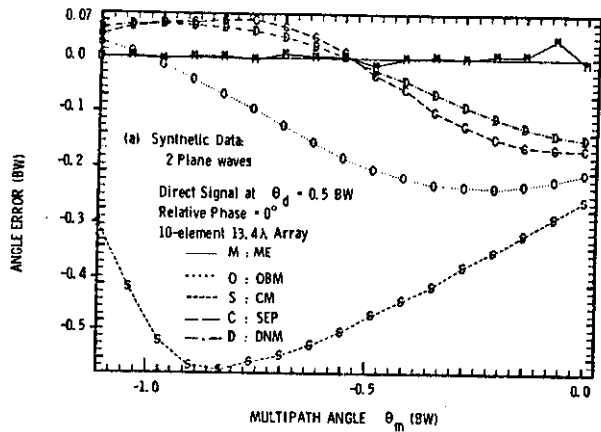


Fig. 2. Synthetic data case.

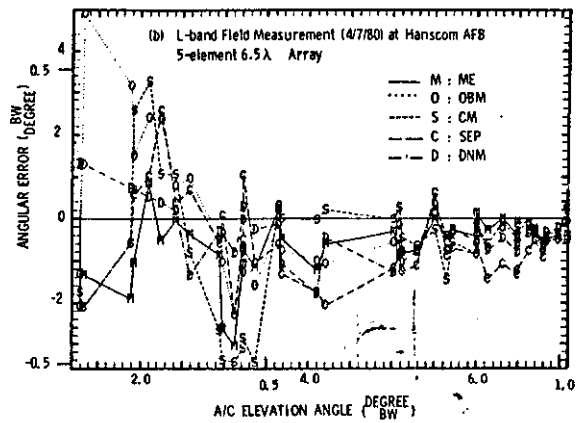
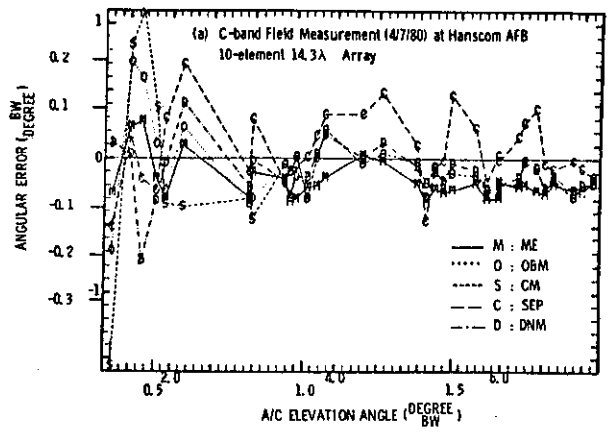


Fig. 3. Hanscom AFB measurement: near-flat terrain.

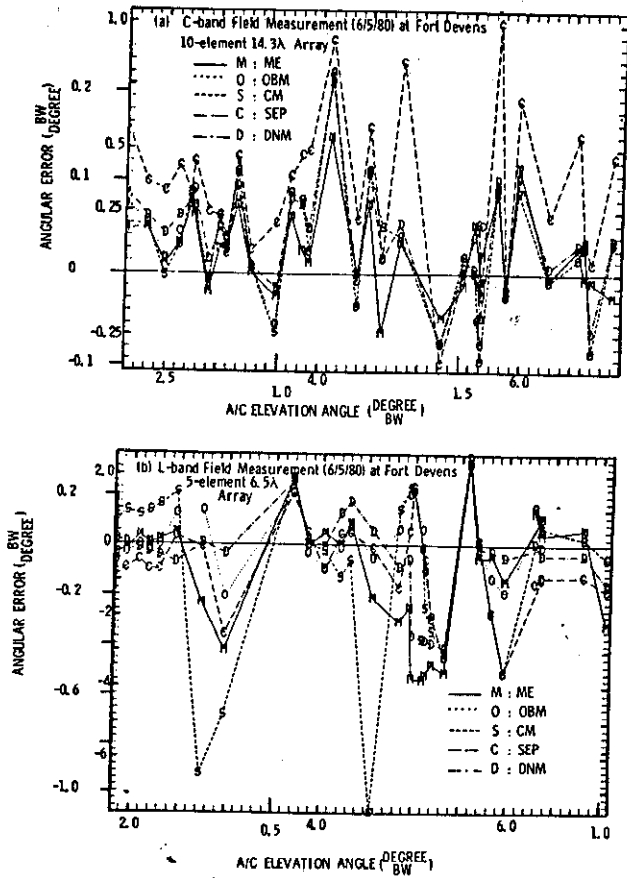


Fig. 4. Fort Devens golf course measurement: rolling terrain.