CLUTTER FILTER DESIGN FOR MULTIPLE-PRT SIGNALS [1]

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1 INTRODUCTION

The trade-off of range vs. velocity ambiguity is fundamental and operationally significant for many S- and C-band pulsed Doppler weather radars. Transmission schemes using multiple pulse repetition times (PRTs) [i.e., nonuniform pulse spacing] offer the potential for extending the unambiguous measurement range by resolving intervals of velocity ambiguity. Unfortunately, multiple PRT methods can be problematic with low-elevation scanning when ground clutter removal is required. We have constructed both Chebyshev and mean-squared error (MSE) design algorithms (Chornoboy, 1993) that deal with design in the complex domain; the MSE algorithms are described below.

2 DESIGN ISSUES

Consider the design of a finite impulse response filter. For an N-coefficient filter, as many as K designs may be required, where K is the number of distinct pulse arrangements of length N. Since it is easier to consider a specific example, we focus primarily on the simple case shown in Fig. 1, that of an alternating-PRT scheme. Here two filters are "multiplexed" in the sense that one set of coefficients[2] $H_1 = [h_{10} \cdots h_{1\,N-1}]^T$ operates on sequences beginning with the longer pulse interval T_1, and a second set (H_2) operates on sequences beginning with T_2.

Velocity estimates for the alternating-PRT signal of Fig. 1 can be obtained by using the single-lag autocorrelation method known as Pulse Pair. If $\hat{R}_1 = \hat{R}(T_1)$ represents an autocorrelation estimate obtained from pulses separated by T_1, then the Doppler velocity (shift) can be estimated as $\hat{V}_1 = -(\lambda/4\pi T_1)\arg(\hat{R}_1)$, where λ is the radar wavelength. Similarly, estimates $\hat{R}_2$ and $\hat{V}_2$ can be obtained corresponding to the interval T_2.

The estimates $\hat{V}_1$ and $\hat{V}_2$ "fold" at values of V

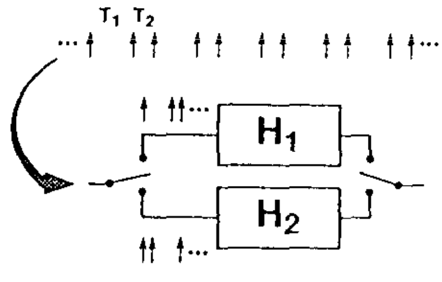

Figure 1: Alternating-PRT Filtering Scheme.

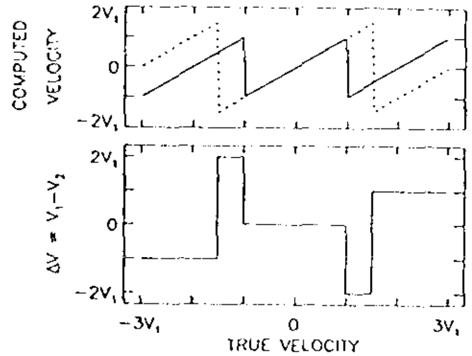

Figure 2: Ideal Dealiasing Using ΔV Method.

determined by the intervals T_1 and T_2. If $T_1 \neq T_2$, velocity folding can be detected and corrected by examining the phase difference between $\hat{R}_1$ and $\hat{R}_2$. This can either be done using the "difference" $\arg(\hat{R}_1\hat{R}_2^*)$, as discussed by Zrnić and Mahapatra (1985), or the difference $\hat{V}_1 - \hat{V}_2$, as considered by Sirmans et al. (1976). The latter is illustrated in Fig. 2, which plots ideal relationships for the case $2T_1 = 3T_2$. The upper plot shows the folding patterns for $\hat{V}_1$ and $\hat{V}_2$ as functions of true velocity. The lower plot illustrates the behavior of the difference $\Delta V = \hat{V}_1 - \hat{V}_2$, and to the extent that this quantity has a unique mapping to intervals of true velocity, ambiguities can be resolved.

It is desirable to have a magnitude response over the extended velocity interval that is free of "blind speeds" so that measurements $\hat{V}_1$ and $\hat{V}_2$ will always be available. However, because $\hat{R}$ phase is key to velocity estimation and ambiguity resolution, it too is an important aspect of the filter design process. Figure 3 shows a staggered-

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[2] Throughout, superscript "T" is used to represent matrix transpose; "$*$", complex conjugate; and "$\dagger$", conjugate transpose.

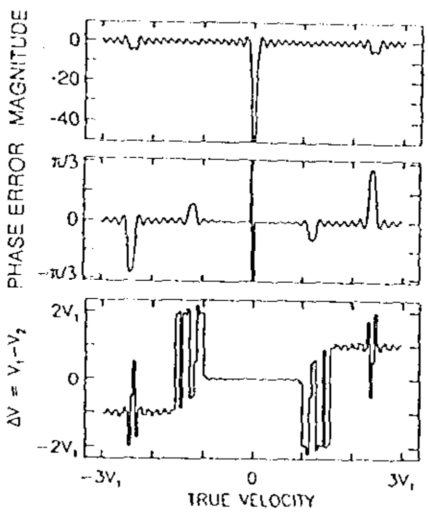

Figure 3: Filter Response and ΔV Transfer Function for Staggered Design Without Phase Control.

PRT design similar to that achievable by adapting uniform sampling methods [Banjanin and Zrnić (1991)]. For an assumed 3:2 stagger spacing, a 33-coefficient filter was designed to provide a stopband half width equaling 0.04 V_1. Although a near "flat" magnitude response has been obtained (top), there are intervals where the phase response deviates significantly from linear phase (middle), and the effect of this phase error on the ideal ΔV transfer function (bottom) is near devastating. Banjanin and Zrnić (1991) have described a method that would work around those areas where the ΔV measure is most impaired.

3 MSE DESIGN EQUATIONS

The frequency response of filter H_k is defined by

$$\mathcal{H}_k = \mathcal{H}_k(\omega) \stackrel{\text{def}}{=} \sum_{n=0}^{N-1} h_{kn}\, e^{j\omega\tau_n} \;, \qquad (1)$$

where τ_n is the time of the n^{th} input sample (relative to the filter start). Let $\mathcal{D}_k = \mathcal{D}_k(\omega)$ represent the desired output response. The filter design problem is to find coefficients H_k that best fit functions $e^{j\omega\tau_n}$ to the desired frequency response $\mathcal{D}_k$.

It is straightforward to set up MSE design functionals by taking M discrete frequency samples of $\mathcal{H}_k$ and $\mathcal{D}_k$. First, define $M \times N$ real-valued matrices C and S, where C is given by

$$C = \begin{bmatrix} \cos\omega_0\tau_0 & \cdots & \cos\omega_0\tau_{N-1} \\ \vdots & & \vdots \\ \cos\omega_{M-1}\tau_0 & \cdots & \cos\omega_{M-1}\tau_{N-1} \end{bmatrix}, \quad (2)$$

and S is defined similarly using "sin" instead of "cos". Second, let σ_k represent the desired output-sample times (i.e., group delay) for the filters, and construct output vectors $\tilde{C}_k$ and $\tilde{S}_k$, where $\tilde{C}_k = [\cos\omega_0\sigma_k \cdots \cos\omega_{M-1}\sigma_k]^T$, etc. Finally, group "sin" and "cos" terms together to form complex input $\Psi = C + jS$ and output $\tilde{\Psi}_k = \tilde{C}_k + j\tilde{S}_k$, and specify the desired magnitude response via an $M \times M$ diagonal matrix D. The ideal output response for filter k is $D\tilde{\Psi}_k$; the approximation to this is ΨH_k, and the approximation error is $\mathcal{E} = \Psi H_k - D\tilde{\Psi}_k$. The weighted squared error $\mathcal{E}^\dagger Q \mathcal{E}$ can be minimized to obtain an MSE solution for H_k. The $M \times M$ real-valued matrix Q is used to introduce relative weighting for pass-, transition-, and stop-band regions. We have found it useful also to place a constraint on the maximum gain of the filter, which can be included via a term such as $H_k^T H_k$.

Although the above provides a design based on minimizing the complex-domain error, it may not be sufficient because it does not permit independent control of phase vs. magnitude error. Let $\epsilon_k = \epsilon_k(\omega) = \omega\sigma_k - \angle\mathcal{H}_k$ represent the phase error for filter k. For $|\epsilon_k| < \pi/2$, the trigonometric inequality

$$|\epsilon_k| \le \frac{\pi}{2}\,|\sin(\omega\sigma_k - \angle\mathcal{H}_k)| \qquad (3)$$

holds, and where $|\mathcal{H}_k| \ne 0$,

$$\sin\angle\mathcal{H}_k = \frac{1}{|\mathcal{H}_k|}\sum_n h_{kn}\sin\omega\tau_n \;. \qquad (4)$$

Equations 3 and 4 can be combined to yield the phase-error constraint

$$|\epsilon_k| \le \frac{\pi}{2\,|\mathcal{H}_k|}\left|\sum_n h_{kn}\sin\omega(\sigma_k - \tau_n)\right| , \qquad (5)$$

which can be used to force $|\epsilon_k|$ small, to the extent that $|\mathcal{H}_k|$ cooperates. Define the $M \times N$ matrix $\Theta_k =$

$$\begin{bmatrix} \sin\omega_0(\sigma_k - \tau_0) & \cdots & \sin\omega_0(\sigma_k - \tau_{N-1}) \\ \vdots & & \vdots \\ \sin\omega_{M-1}(\sigma_k - \tau_0) & \cdots & \sin\omega_{M-1}(\sigma_k - \tau_{N-1}) \end{bmatrix},$$

and form squared error term $\Phi_k^T P \Phi_k$, where $\Phi_k = \Theta_k H_k$ and P is an optional weighting matrix for the phase error.

An error functional for the phase-controlled design can be written

$$f(H_k) = \mathcal{E}^\dagger Q \mathcal{E} + \Phi_k^T P \Phi_k + H_k^T H_k \;. \qquad (6)$$

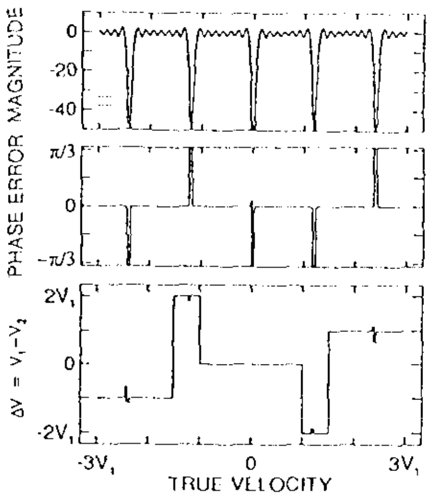

Figure 4: Filter Response and ΔV Transfer Function for Staggered Design With Excessive Phase Control.

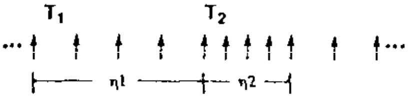

Figure 5: A Block-Staggered Sampling Scheme

This is quadratic in H_k and has solution

$$H_k = [\Re e(\Psi^\dagger Q \Psi) + \Theta_k^T P \Theta_k + I]^{-1} [\Re e(\Psi^\dagger Q D \hat{\Psi}_k)]$$
$$= [C^T Q C + S^T Q S + \Theta_k^T P \Theta_k + I]^{-1} \cdot [C^T Q D \tilde{C}_k + S^T Q D \tilde{S}_k]. \quad (7)$$

Note that this solution only requires real-domain computation.

At the extreme $P = \emptyset$ (i.e., no phase control), the design of Fig. 3 results. If P is instead set very large, placing high priority on a linear phase response, the design of Fig. 4 results. This second design approaches the "split uniform PRT" filter discussed by Banjanin and Zrnić (1991). Linear phase is obtained at the expense of introducing notches (blind speeds) in the magnitude response. For the alternating-PRT signal, magnitude response blind/dim speeds vs. nonlinear phase is a fundamental trade-off which no optimal design can fully overcome.

An additional "design" option exists however for weather-radar application because coherent averaging over many intervals T_1 (T_2) is typically employed. A simple extension to the design illustrates one practical potential. Consider the block-staggered signal structure illustrated in Fig. 5. Pulse spacing T_1 is repeated η_1 times, followed by interval T_2 η_2 times; after which the pattern repeats. This signal requires $K = \eta_1 + \eta_2$ filter coefficient sets. As with the above alternating-PRT example, it is unlikely that exact linear phase can be achieved without some compromise in magnitude response. However, the added complexity of the pulse pattern enables an improved balance between magnitude and phase response. Furthermore, since there is variety in the filters affecting T_1 (T_2) intervals (across the confines of one block), phase and magnitude responses can be balanced among filters by "dithering" (distributing) the error. Very satisfactory response profiles can result as shown in Fig. 6, which shows the results for a (4,4) block-stagger design using a phase-control weighting intermediate to that used in Figs. 3 and 4.

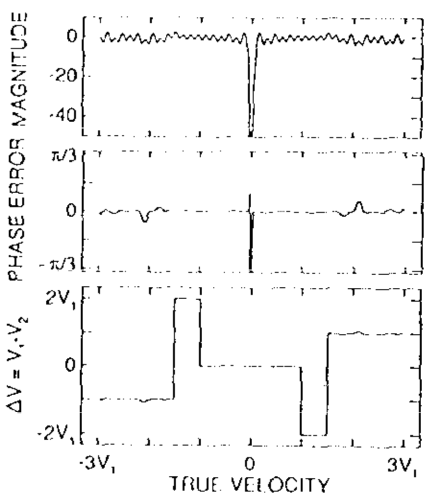

Figure 6: Filter Response and ΔV Transfer Function for Block-Staggered Design With Phase Control.

References

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