

**Project Report  
ATC-204**

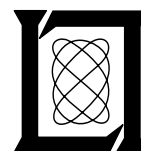
# **SGS85-WGS84 Transformation: Interim Results**

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**24 August 1993**

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Prepared for the Federal Aviation Administration,  
Washington, D.C. 20591

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1. Report No. ATC-204	2. Government Accession No. DOT/FAA/RD-93/29	3. Recipient's Catalog No.	
4. Title and Subtitle SGS85-WGS84 Transformation: Interim Results		5. Report Date 24 August 1993	
		6. Performing Organization Code	
7. Author(s) Pratap N. Misra, Richard I. Abbot, and Edward T. Bayliss		8. Performing Organization Report No. ATC-204	
9. Performing Organization Name and Address Lincoln Laboratory, MIT P.O. Box 73 Lexington, MA 02173-9108		10. Work Unit No. (TRAIS)	
		11. Contract or Grant No. DTFA-01-89-Z-02030	
12. Sponsoring Agency Name and Address Department of Transportation Federal Aviation Administration Systems Research and Development Service Washington, DC 20591		13. Type of Report and Period Covered Project Report	
		14. Sponsoring Agency Code	
15. Supplementary Notes  This report is based on studies performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology. The work was sponsored by the Air Force under Contract F19628-90-C-0002.			
16. Abstract  GPS and GLONASS employ different geocentric Cartesian coordinate frames to express the positions of their satellites and, therefore, of their users. GPS uses WGS84; GLONASS, SGS85. Interest in the civil aviation community in using signals from both systems requires that a transformation between the two coordinate frames be determined. We present an estimate of the SGS85-WGS84 transformation.			
17. Key Words GPS GLONASS WGS84 SGS85  satellite navigation geocentric coordinate frames similarity transformation		18. Distribution Statement  This document is available to the public through the National Technical Information Service, Springfield, VA 22161.	
19. Security Classif. (of this report)  Unclassified	20. Security Classif. (of this page)  Unclassified	21. No. of Pages  38	22. Price



## **ACKNOWLEDGMENTS**

The authors are grateful to Joseph F. Dorfler, Program Manager, FAA Satellite Program, for his support of this work. We are also grateful to Dr. M. Kumar, DMA System Center, for several helpful discussions. Thanks are also extended to our Lincoln colleagues Dr. R. Muchnik and Dr. M. Pratt for technical assistance, and to K. Krozel for editorial assistance.



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# 1. INTRODUCTION

GPS and GLONASS, the navigation satellite systems being deployed by the U.S. and Russia, respectively, are approaching completion of their developmental phases. GPS is planned to become operational in 1995; GLONASS is expected to follow. A large and evolving community of users of these systems includes the aviation industry. The civil aviation industry, in particular, sees an extraordinary potential in an integrated use of the two systems to enhance both economy and safety. The impetus for the integrated-use comes from the following consideration. The number of satellites in either constellation is not adequate to allow a self-contained integrity check on the measurements, but the combined measurements offer enough redundancy to permit receiver autonomous integrity monitoring to detect and isolate anomalous measurements. The integrated use is seen to be a potential sole-means navigation system for all phases of flight, including precision approach and landing. An FAA-sponsored program has been underway at Lincoln Laboratory to examine the technical issues associated with such integrated use and to establish the performance achievable therefrom.

GPS and GLONASS are autonomous systems, each with its own time reference and spatial coordinate system. Before measurements from the two systems can be combined, a connection needs to be established between the two time scales and the coordinate frames. The time scale issue can be resolved without an external means at worst by 'sacrificing' a measurement to estimate the instantaneous bias between the two scales. The coordinate frame issue is not resolved so simply. GPS specifies the positions of its satellites in an earth-centered, earth-fixed (ECEF) Cartesian coordinate frame WGS84 [1]; GLONASS in SGS85 [2]. Actually, the geocentric coordinate frame, specified as an abstract mathematical entity, is just one element of an overall reference system to represent the earth from geometric, gravitational, and geodetic standpoints. Like its counterpart, WGS84, SGS85 is a self-contained and self-consistent system within which to define a 3-D position. The values of the defining parameters of SGS85 gravity model, and the ellipsoid are slightly different from those of WGS84. These differences, however, are easily accommodated. A more difficult problem arises from the differences in the ECEF coordinate frames. It is this problem we address here. It should be noted that our immediate objective is to obtain a solution consistent with the requirements of civil aviation, rather than those of geodesy.

The ECEF coordinate frames of WGS84 and SGS85 differ in their formal definitions [1, 2]. While each locates the origin at the center of mass of the earth, the directions of the z-axis are different: WGS84 defines it as passing through the instantaneous pole of 1984.0; SGS85 adopts instead the average position of the pole between the years 1900 and 1905. This description, however, is not adequate as a basis for determining a transformation. Actually, even if the formal definitions were identical, it would not have ensured that the coordinates of a point as determined by measurements from the two systems will be identical. The coordinate frame for each system is realized (or, implemented) by adopting the coordinates of a set of stations. A consistent set of such coordinates defines implicitly the ECEF coordinate frame (i.e., an origin, a set of directions for the Cartesian axes, and a scale factor). Therefore, even if GPS and GLONASS had

adopted the same definition for the reference coordinate frame, the autonomous implementation by each system would have kept the two from being identical.

We review in the next section how a transformation between two Cartesian coordinate frames is specified, and how its parameters are estimated in general. Section 3 presents our approach to estimation of WGS84 - SGS85 transformation, and describes our data set. Section 4 gives the results of data analysis and the preliminary estimates of the parameters.

## 2. COORDINATE TRANSFORMATION: APPROACH

The coordinates of a point in one Cartesian coordinate frame may be related to those in another via a similarity transformation involving seven parameters corresponding to a translation of the origin, rotations of the three axes, and a scale factor [3, 4]. Specifically, a Cartesian coordinate system  $W: (x,y,z)$  is related to the coordinate system  $S: (u,v,w)$ , nearly-aligned to it, by (Figure 1):

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} + (1 + \delta s) \begin{bmatrix} 1 & \omega & -\phi \\ -\omega & 1 & \epsilon \\ \phi & -\epsilon & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad (2.1)$$

where  $(\Delta x, \Delta y, \Delta z)$ : coordinates of the origin of the frame  $(u,v,w)$  in the frame  $(x,y,z)$

$\epsilon, \phi, \omega$ : differential rotations (in radians) around the axes  $(u, v, w)$ , respectively, to establish parallelism with the  $(x, y, z)$  frame; and

$\delta s$ : differential scale change.

The values of these parameters can be estimated simply if we knew the positions of a set of points as expressed in both coordinate frames. There are seven unknowns, and, therefore, we need at a minimum coordinates of three points in both coordinate frames for an estimate of the parameter set, though not necessarily a good one. It is clear *a priori* that the quality of the parameter estimates will depend upon (i) the number of such points and their spatial distribution, and (ii) the quality of the position estimates.

Let  $(x_i, y_i, z_i)$  and  $(u_i, v_i, w_i)$  be the coordinates of the  $i$ th point in the two coordinate frames,  $i = 1, 2, \dots, n$ . Let  $P = \{\Delta x, \Delta y, \Delta z, \delta s, \epsilon, \phi, \omega\}$  be a set of parameter values, and let  $r_i(P)$  be the corresponding residual vector defined as the difference between vectors  $(x_i, y_i, z_i)$  and the transformed vector  $(u_i, v_i, w_i)$ .

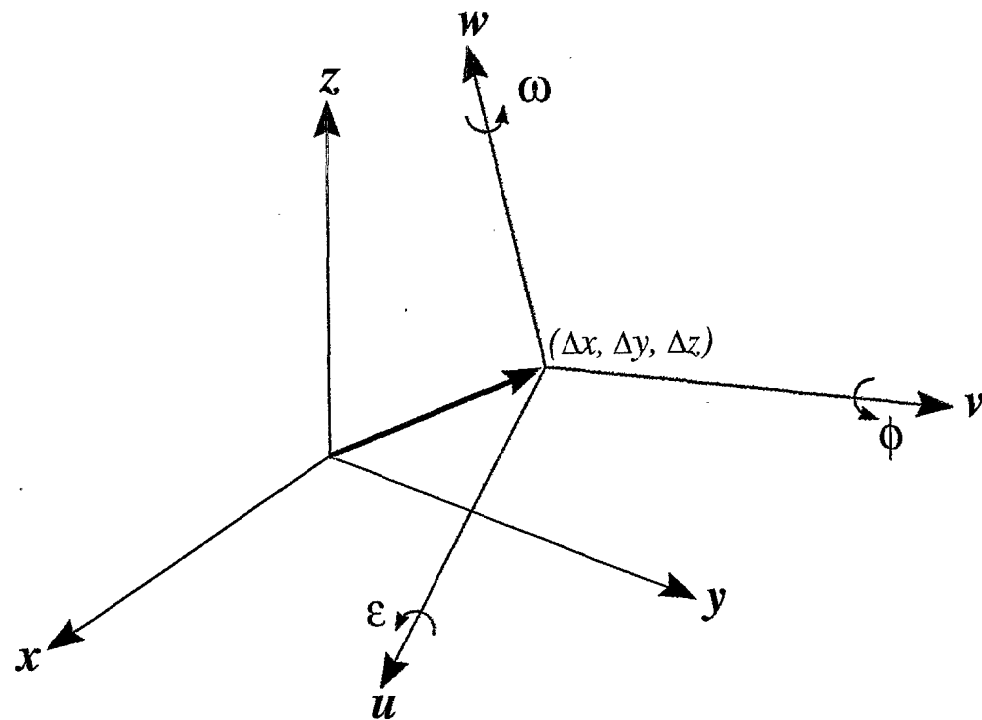
$$r_i(P) = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} - \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} - (1 + \delta s) \begin{bmatrix} 1 & \omega & -\phi \\ -\omega & 1 & \epsilon \\ \phi & -\epsilon & 1 \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix}, \quad (2.2)$$

The problem of estimation of the parameter set  $P$  in (2.2) is typically formulated as one of least squares: Choose parameter values which minimize the sum of the squared differences (post-fit residuals) between the coordinates:

$$\text{Min}_P \sum_i \left\| r_i(P) \right\|^2,$$

where  $\|\cdot\|$  represents the length (Euclidean norm) of a vector. Any *a priori* knowledge of the disparity in the quality of the data can be accounted for in terms of relative weights assigned to the different measurements, or their components. Let  $W_i$  be a diagonal matrix whose entries reflect these relative weights, then the above minimization problem can be rewritten as:





$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} + (1 + \delta s) \begin{bmatrix} 1 & \omega & -\phi \\ -\omega & 1 & \epsilon \\ \phi & -\epsilon & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

**TRANSFORMATION PARAMETERS:**

$$\{\Delta x, \Delta y, \Delta z, \delta s, \epsilon, \phi, \omega\}$$

Figure 1. Similarity transformation.





$$\text{Min}_P \sum_i \left\| W_i r_i(P) \right\|^2$$

The challenge in estimation of the SGS85 - WGS84 transformation so far has been in getting the position estimates of a common set of points in *both* coordinate frames. Actually, getting accurate position estimates in WGS84 using GPS measurements is now straightforward and common. There exists a global network of well-known benchmark points with precisely-defined positions in WGS84. The position of a point can, therefore, be obtained accurately by determining its position precisely relative to one or more of these benchmarks. Getting accurate position estimates in SGS85, however, continues to be a problem. There are two main reasons. First, we do not know of a network of benchmark points with well-defined positions in SGS85, and must, therefore, resort to point positioning using GLONASS measurements, a less precise scheme. Secondly, GLONASS receivers have been a rare commodity so far, making it difficult to take measurements at many points. Both problems will be overcome in time. For now, we address the problem by taking advantage of our unique resources: GLONASS receivers in our SatNav Laboratory, developed under the sponsorship of the FAA; and a deep-space tracking network operated in part by MIT for the U. S. Air Force.

For a common set of points, we have chosen the positions of GLONASS satellites. The main reason for this choice is that it gets around the problem of position estimation in SGS85. The satellites broadcast their coordinates in SGS85, and anyone with access to a GLONASS receiver can obtain these. Now the problem shifts to obtaining the position estimates of GLONASS satellites in WGS84; that is where the deep-space tracking network comes in. The position estimates in WGS84 are obtained by tracking GLONASS satellites; fitting an orbit to the measurements; and sampling the orbit (Figure 2). The characteristics of these position estimates, and estimation of the transformation parameters based on them, are discussed in Sections 3 and 4.

To our knowledge, the first attempt at estimation of this transformation was made in a joint exercise by Honeywell, Northwest Airlines, and the erstwhile Leningrad Radiotechnical Research Institute (LRRI) [5]. While this endeavor had the benefit of being able to take measurements all over the globe, the GLONASS receiver available for the exercise, LRRI's ASN-16, lacked precision. Secondly, the positioning results provided by this receiver were not in SGS85, as assumed by the experimenters. The results, therefore, were not useful.



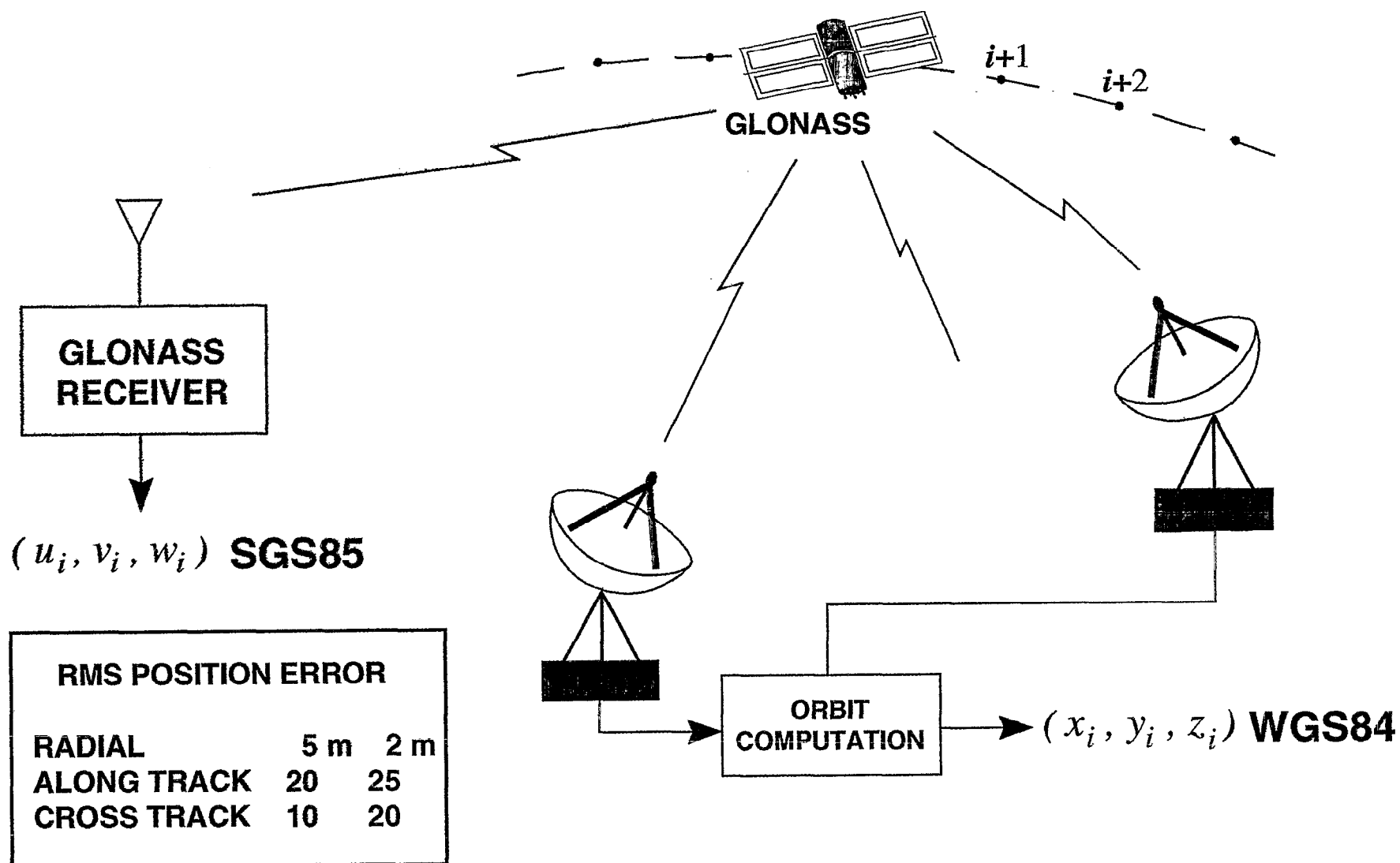


Figure 2. Position estimates in SGS85 and WGS84.



### **3. GLONASS EPHEMERIDES IN SGS85 & WGS84**

#### **3.1 GLONASS EPHEMERIDES IN SGS85**

The GLONASS satellites broadcast their positions in SGS85 as a part of their navigation messages. We have been monitoring these signals nearly continuously since 1990 using two laboratory-quality experimental receivers. The results on GLONASS performance based on these measurements have been reported previously [6, 7]. During our period of observation, the navigation message broadcast by each satellite changed mostly on the half-hour, each specifying the position of the satellite at the mid-point of the next half-hour interval. The navigation message also includes other parameters (e.g., velocity and acceleration vectors, clock corrections, etc.) to permit a user to compute the satellite position at another instant. For our purposes, we have chosen to work with the position vectors in SGS85 as given each half-hour by GLONASS.

The GLONASS ICD [2] specifies the rms errors in the position estimates carried in the navigation messages as follows: 5 m radially, 20 m along track, and 10 m cross track (rms position error: 23 m). While we have no direct verification of these numbers, our experience in position estimation with GLONASS is consistent with them [6, 7]. Some remarks are in order on the quality of these position estimates, and its implications for the estimation of the transformation parameters. As noted previously, the better the position estimates, the better the estimates of the transformation parameters based on them. The position estimates provided by GLONASS of its satellite positions are actually better than they might first appear, at least for the purpose of estimation of the rotational and scale parameters of a coordinate transformation. A perspective on these specifications is obtained by noting that the points are at four earth radii from the origin, and the effect of a rotation is magnified four fold over that on the earth. If we were limited to position estimates of points on the earth, the estimates would have been required to be much more accurate in order to be comparable in quality to our data set. Specifically, the horizontal components of the error would have been required to be no more than one-fourth the size of the cross-track and along-track errors above: rms horizontal error < 6 m. And we would indeed be hard-pressed to match that in point positioning with GLONASS today.

#### **3.2 GLONASS EPHEMERIDES IN WGS84**

Estimation of the GLONASS satellite positions in WGS84 is based on the measurements in the deep-space tracking network metric database. For our experiments, the GLONASS tracking data were obtained from two radar systems: Millstone radar in Westford, MA, and ALTAIR radar in Kwajalein, Marshall Islands. Tracking data were also obtained from a number of optical sites from around the world, including the four Ground-based Electro-optical Deep Space Surveillance (GEODSS) sites, and the Maui Optical Tracking and Identification Facility (MOTIF) in Hawaii. These are precise and highly-calibrated instruments. The radars provide measurements of azimuth, elevation, range, and range rate. The optical sites provide measurements of the right ascension and declination. The measurement accuracies of the two radar systems and the optical sites are summarized in Table 1 below.

**Table 1. Measurement Errors**

	<b>Azimuth</b>	<b>Elevation</b>	<b>Range</b>	<b>Range rate</b>
Millstone	5 mdeg	5 mdeg	1-2 m	1 mm/s
ALTAIR	15	15	15	20
	<b>Right Ascension / Declination</b>			
GEODSS	2-3 mdeg			
MOTIF	1-2			

Typically, we had one set of measurements from each sensor over a period of several minutes at some point in each pass of a satellite. A data collection period lasted 10-20 days. The limiting factor in the precision of the orbits has been the amount of tracking data available. Determination of more precise orbits would require measurements from each sensor of longer duration while a satellite was at several different elevations. We expect to have access to more measurements and, therefore, more precise orbits in a future experiment.

The orbit determination was done with high-precision orbit analysis software using a least-squares approach, with the tracking measurements weighted to reflect their *a priori* measurement uncertainties. Typically about two weeks of tracking data were used for each orbit determination. Once the orbital parameters were estimated, the software generated the state vectors at 30-minute intervals corresponding to the epochs of the GLONASS broadcast ephemeris.

The quality of an orbit is characterized by comparing it with a set of tracking measurements not used in the fit. There are a number of ways to do this. Our approach consists of withholding the tracking measurements from one pass of a satellite in the orbital fit process, and then comparing these measurements with the fitted orbit. This step is repeated, now data from a different pass withheld. The disagreement, characterized in rms terms, reflects the errors in both the measurements and the fitted orbit. Since we know the measurement errors *a priori*, as given above, the orbital errors can be extracted. On the basis of such an analysis, the rms orbital errors in our experiments have been estimated to be as follows: 1-2 m radially, 25-30 m along track, and 20-25 m cross track (rms position error: 30-40 m). The quality of our position estimates in WGS84 is similar to that in SGS85, and the earlier remarks regarding the accuracy requirements of position estimates on earth in order to be comparable apply here also.

## **4. SGS85 - WGS84 TRANSFORMATION: PARAMETER ESTIMATION**

### **4.1 DATA SET**

We have run three data collection experiments in 1991-1992 for the purpose of estimation of SGS85 - WGS84 transformation. Each experiment lasted 10-20 days and consisted of recording broadcasts from two to four GLONASS satellites while in view at Lincoln Laboratory, and obtaining tracking data for them simultaneously from the deep-space tracking network. The common set of points consists of the positions of a GLONASS satellite at half-hour intervals while in view at Lincoln Laboratory. Typically a satellite is visible for 6-8 hours a day, and the common set of points has consisted of 150 or more points per satellite. A GLONASS satellite repeats its ground track on the ninth day, so these points are well-distributed in space. The fitting of a precise orbit to the tracking measurements available, however, has remained the weaker link in this process, and the quality of orbital fit has varied. But the basic conclusion on parameter estimates from each of these data sets is substantially the same.

We present here data for two GLONASS satellites from the January 1992 experiment. Data Set 1 corresponds to the satellite with Cosmos number 2111 (frequency channel 19), and Data Set 2 to that with Cosmos number 2141 (frequency channel 14). The analysis is based on the position estimates in the two coordinate frames at common time instants: quarter-to and quarter-after each hour. Strictly speaking, the two sets of measurements use different time scales: The position estimates in SGS85 correspond to time instants as defined in UTC(SU); the positions in WGS84 are defined in accordance with UTC(USNO). But the difference between the two time scales is about 4 microseconds for the period in question [8], and no correction was deemed necessary to account for this difference. As will be seen later, we have also taken advantage of our knowledge of the velocity vector at each point to examine the components of the error in terms of its radial, along-track, and cross-track components. The data corresponding to the two satellites have been analyzed separately, each leading to basically the same conclusions.

### **4.2 PARAMETER ESTIMATION**

First we compare the two sets of coordinates for the common set of points to assess the 'size' of their differences. For each point we take the difference of its geocentric coordinates in WGS84 and SGS85 and express this difference vector in terms of its radial, along-track and cross-track components. The rms values for these differences for the two data sets are: 2-3 m radially, 30 m along track, and 60-85 m cross track.

The first issue we address is: How much of the observed differences in the position estimates in the two coordinate frames can be attributed to the errors in them? Stated differently, if the two coordinate frames were indeed identical and the position errors were as given earlier, how large a difference might we see between the position estimates of a point in each? The answer is straightforward, given the rms position errors in SGS85 (5 m radially, 20 m along track, 10 m cross track), and those in WGS84 (1 m radially, 25 m along track, and 20 m cross track). The rms values of the differences are: 5 m radially,

32 m along track, and 22 m cross track (rss  $\approx$  40 m). Comparing these values to the observed differences, we note first that the size of the radial error in the position estimates in GLONASS broadcasts is, obviously, considerably less than specified. Secondly, it is clear that the radial and along-track components of the coordinate differences may be explained entirely by the errors in the two measurement sets. The cross-track error of 60-80 m, however, is much larger than can be so explained. We, therefore, conclude that the two coordinate frames have differences.

The next question we address is: How much additional error will be introduced in position estimation if the coordinate differences were to be disregarded in combining measurements from GPS and GLONASS? From the differences in the coordinates in the two systems cited above, for a point located in the GLONASS orbit this error may be 80 m rms, mostly in the cross-track direction. For points located at one-fourth the distance to the GLONASS orbit (i. e., on or near the earth), most of the error will be in the horizontal plane, amounting up to 20 m rms. From the point of view of civil aviation, an error this size is not significant in en route and terminal phases of flight and in nonprecision approaches, and a transformation may be dispensed with. Actually, the differences between the two coordinate frames may also be disregarded in precision approaches carried out in a differential mode: the error so introduced will be common to the reference station and the approaching aircraft, and will cancel.

Consider next the estimation of the transformation parameters. The approach is weighted least-squares to account for the known disparity in the size of the errors in the radial, along-track, and cross-track components of the position vectors. The approach is implemented by computing at each point the difference vector  $\mathbf{r}_i(P)$ , as defined in (2.2), and a rotation matrix  $R_i$  to transform  $\mathbf{r}_i$  to get its components in the radial, along-track and cross-track directions. Given the disparity in the quality of the position estimates in these directions, these squared residuals are weighted by the inverse of the corresponding mean-squared error in their estimates (radially: 1, along track: 0.06; cross track: 0.25). These weights are incorporated in our formulation via diagonal matrix  $W$  whose diagonal elements are 1, 0.25, and 0.5. The transformation parameters are estimated so as to minimize the weighted squares summed across all the points.

$$\text{Min}_P \sum_i \left\| W R_i \mathbf{r}_i(P) \right\|^2$$

While weighting plays a role, we have found that small changes in the relative weights have no significant effect on the results.

The results of an exercise in parameter estimation based on the above scheme are given for the two data sets in Table 2. For each data set, Table 2 gives, for different values of the transformation parameters, the corresponding rms values of the radial, along-track and cross-track components of the residual difference between coordinates in SGS85 and WGS84. The first line gives the extent of this difference in the two sets of coordinates without the benefit of a transformation, discussed earlier. These are computed by setting all transformation parameters to zero. The subsequent lines give values of the non-zero transformation parameters, and the corresponding residuals. While we actually minimize a weighted sum of the squared residuals along radial, along-track, and cross-track directions,



their unweighted rss value is given in the last column as the total residual to provide a better 'feel' for the disagreement between the two sets of coordinates. Our objective is to find simple transformation consisting of a minimum number of the parameters. So, we estimate transformations where all but one (two, three, etc.) of the parameters are set to zero, and examine the sizes of the corresponding residuals.

**Table 2. Weighted-Least-Squares Parameter Estimation**

Data Set	Transform Parameters		RMS Residuals (m) (r: radial, a-t: along track; c-t: cross track)			
	Number of Parameters	Estimated Values	r	a-t	c-t	total
1	0		1.9	28.0	61.3	67.4
	1	$\omega = -3.10^{-6}$ rad	1.9	24.2	21.5	32.4
	2	$\omega = -3.10^{-6}$ $\Delta z = 4.1$ m	2.3	24.1	20.7	31.8
	4	$\omega = -3.10^{-6}$ rad $\Delta x = 1.1$ m $\Delta y = 2.0$ $\Delta z = 4.5$	2.7	24.1	20.3	31.6
	7	-----	2.3	25.4	19.2	31.9
2	0		3.1	27.7	83.4	88.0
	1	$\omega = -4.10^{-6}$ rad	3.1	18.6	35.0	39.8
	2	$\omega = -4.10^{-6}$ $\Delta z = 4.5$ m	4.4	18.1	34.0	38.7
	4	$\omega = -4.10^{-6}$ rad $\Delta x = -1.6$ m $\Delta y = 5.2$ $\Delta z = 5.9$	5.3	17.9	32.7	37.7
	7	-----	4.0	25.0	29.9	39.2

We begin by identifying the single most important parameter to characterize the difference between the two coordinate frames. This choice turns out to be simple, clear cut, and consistent across all our data sets: The Parameter  $\omega$  makes a significant impact; the

others, almost none. As is shown in Table 2 (Data Set 1, line 2), a clockwise rotation of the z-axis of SGS85 by  $3.10^{-6}$  rad (0.6") cuts the residuals by half! A similar conclusion is drawn from Data Set 2, though the size of the estimated rotation is slightly larger. The estimates of Parameter  $\omega$  have ranged between  $2.5.10^{-6}$  and  $4.10^{-6}$  rad in our other data sets. For both data sets in Table 2, this rotation reduces the residuals to a level accounted for by the errors in the estimates themselves, as discussed earlier. It is clear that we do not expect the residuals to go down much lower, and, therefore, do not expect much from the remaining transformation parameters.

Actually, a small but consistent reduction in the residuals is observed by including one more parameter in the transformation: A displacement of the origin of SGS85 along the z-axis by about - 4 m. As seen in Table for both data sets, this displacement results in a trade-off between the radial and cross-track components of the residuals, and a slight drop in the total. At this stage, we have squeezed all the information out of the data sets we had to work with. Including the remaining two components of the displacement  $\Delta x$  and  $\Delta y$  in the transformation offers little help in reducing the residuals, and the estimates of these parameters, as obtained from the different data sets, are quite different. The same is true of the full seven-parameter transformation, and, therefore, the corresponding parameter estimates have been left out of Table 2. For both data sets, the total residuals corresponding to the seven-parameter transformation are larger than those for the four-parameter transformation. This, however, is not unexpected since we are minimizing a weighted rss value of the residuals, which is actually smallest for the seven-parameter case.

### 4.3 SGS85 - WGS84 TRANSFORMATION

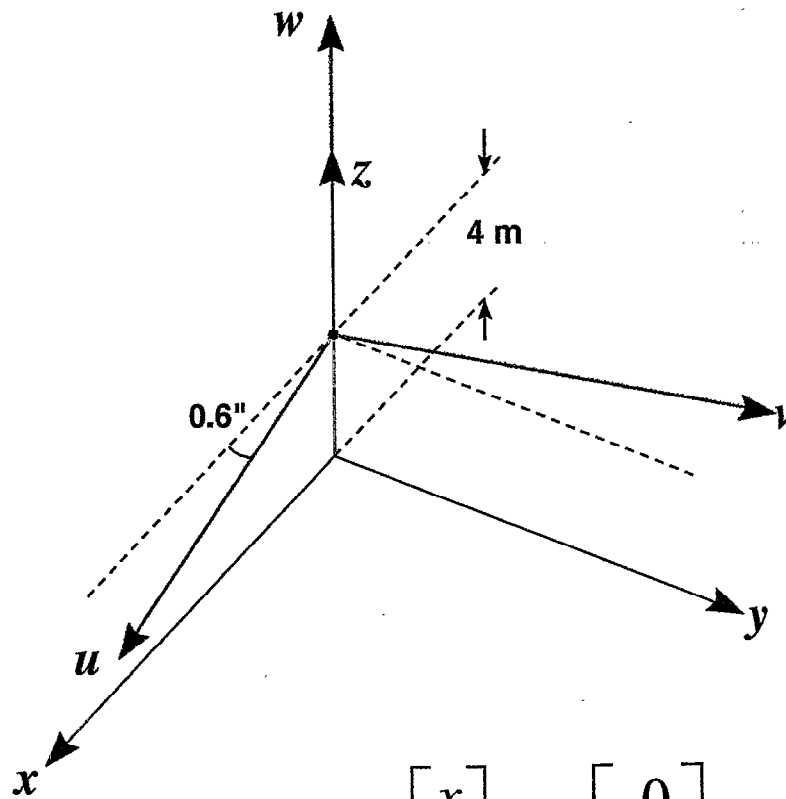
The basic conclusion from Table 2 is that the zero meridian (x-axis) of SGS85 is East of that for WGS84; the equatorial plane of SGS85 is offset North of that of WGS84. A small clockwise rotation (0.6") of the z-axis of SGS85 brings the two coordinate frames substantially into coincidence. The residuals are reduced further, though only slightly, by a 4 m displacement of the origin along its negative z-axis (Figure 3). The remaining five parameters of the transformation are found to have no significant role.

The estimated transformation between WGS84 (W): (x, y, z) and SGS85 (S): (u, v, w) is:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 4 \text{ m} \end{bmatrix} + \begin{bmatrix} 1 & -3.10^{-6} & 0 \\ 3.10^{-6} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad (4.1)$$

We note that aside from the scale factor, this transformation is similar to that between WGS72 and WGS84.

A more detailed look at the two data sets is offered in Figures 4 and 5, showing for each point in the common set the differences in the coordinates along the radial, along-track, and cross-track directions, and the residuals after applying transformation (4.1). As seen earlier in Table 2, the situations radially and along-track are similar for the two data sets, but the cross-track differences are larger for Data Set 2. Apparently the quality of the broadcast ephemeris and/or the fitted orbit for the two data sets are different. The picture



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 4 \text{ m} \end{bmatrix} + \begin{bmatrix} 1 & -3 \cdot 10^{-6} & 0 \\ 3 \cdot 10^{-6} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

**WGS84** **SGS85**

Figure 3. SGS85 - WGS84 transformation.



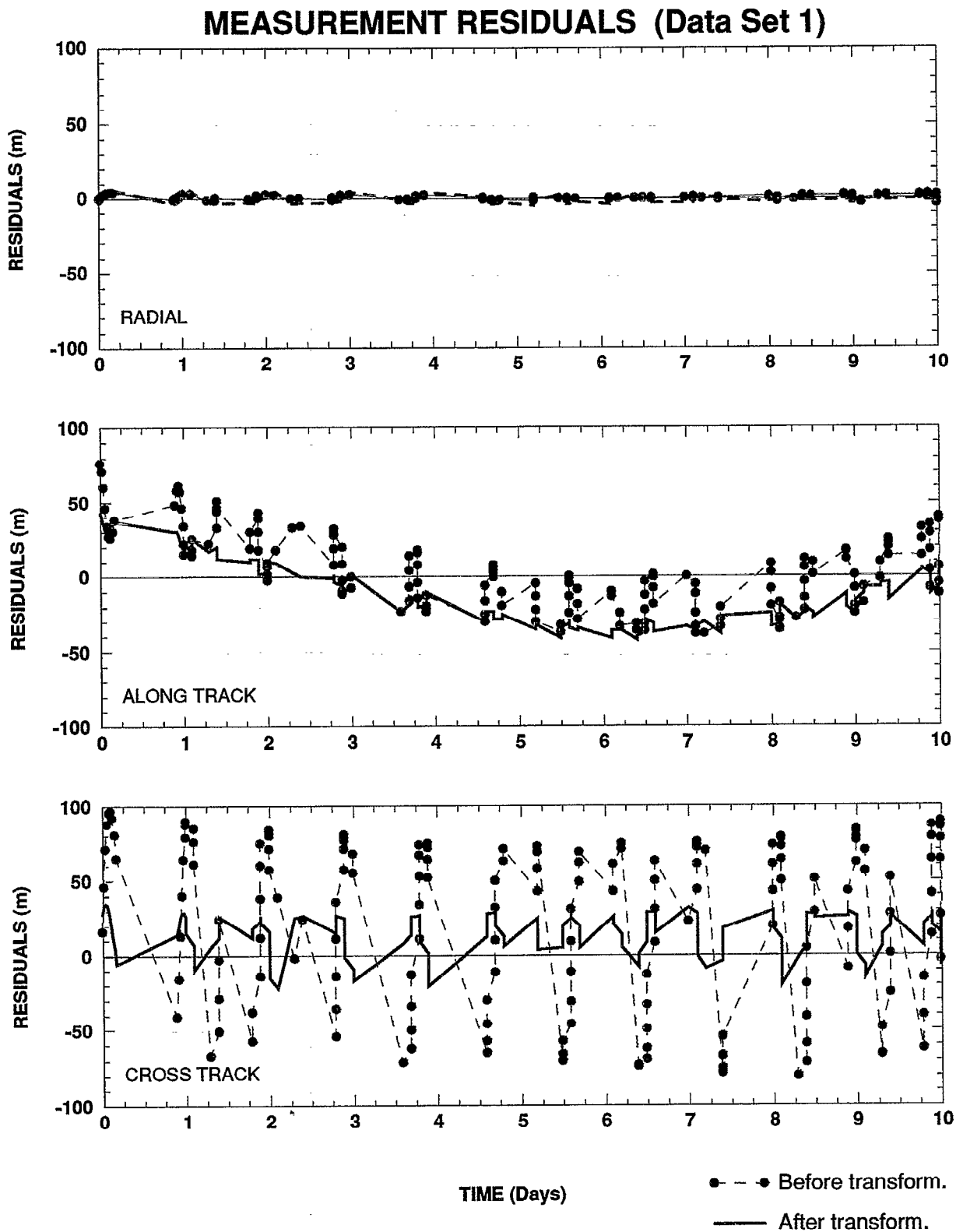


Figure 4. SGS85 - WGS84 transformation - data set 1.



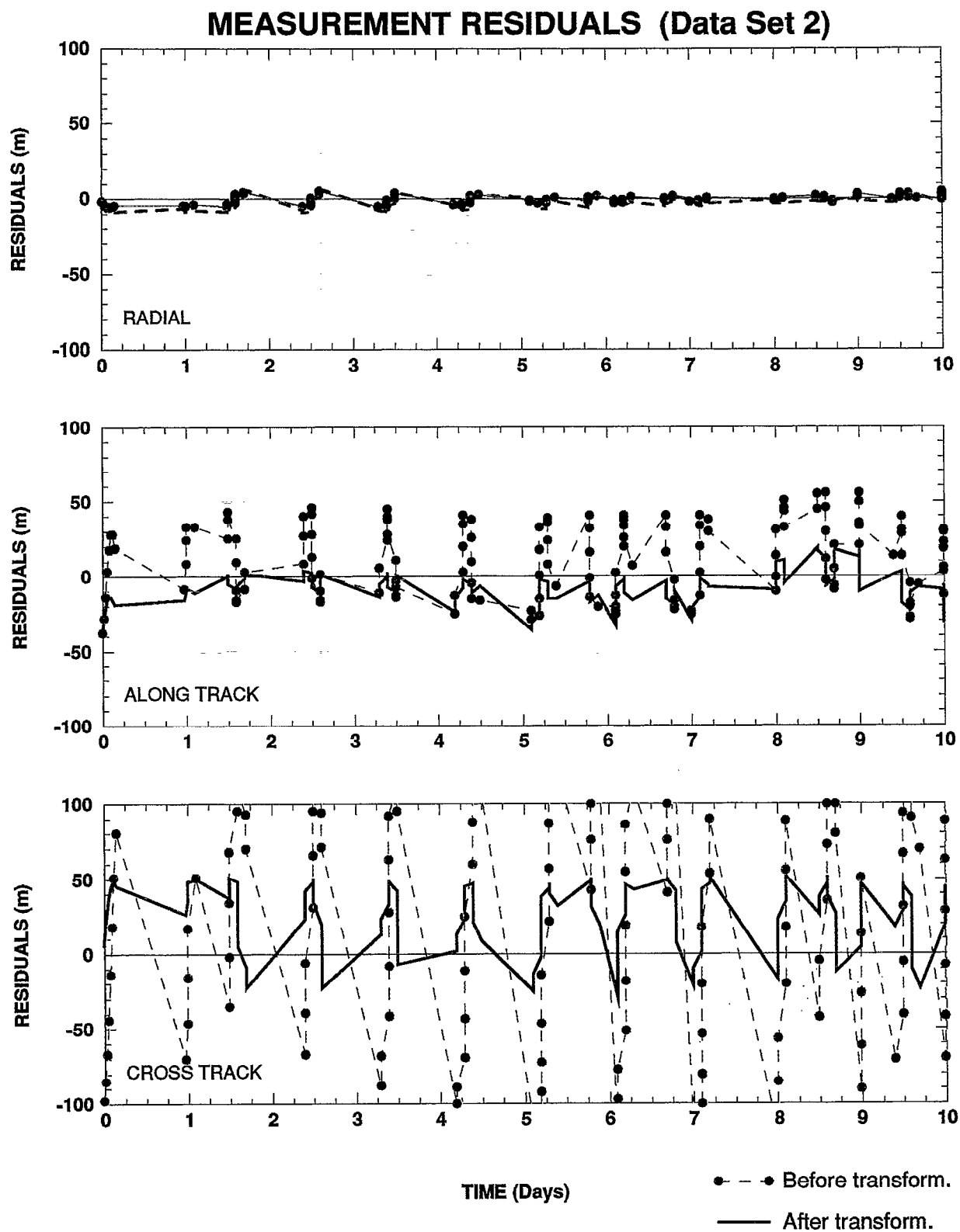


Figure 5. SGS85 - WGS84 transformation - data set 2.





with cross-track residuals improves significantly following the transformation, as also seen earlier in Table 2. The transformation also brings about a small improvement in along-track residuals. For Data Set 1, the coordinate differences along track and the residuals show an apparent quadratic trend, suggesting a systematic error in one or both ephemerides. Finally, the differences in the coordinates in the radial direction were small to begin with, and we did not expect the transformation to produce any significant change.

Ground-based GLONASS measurements at a small set of points confirm both our conclusions regarding the extent of the differences between SGS85 and WGS84, and the validity of the transformation (4.1) between them. As noted earlier, getting accurate position estimates of a set of points in SGS85 is the main difficulty at present in developing a more accurate transformation. Our approach took advantage of the best available position coordinates in SGS85, and estimated the corresponding positions in WGS84. The latter step has room for improvement, and we plan to repeat our experiment with the aim of getting better estimates in WGS84. We do not, however, expect a significant improvement in the transformation until the quality of the position estimates in SGS85 improves.



## 5. CONCLUSIONS

The differences between SGS85 and WGS84 are such that the coordinates of a point on earth differ by under 20 m rms. Our analysis has shown that a small rotation (0.6") of the z-axis brings the two coordinate frames substantially into coincidence. The residuals are reduced further, though only slightly, by a 4 m displacement of the origin along the z-axis. The remaining five parameters of the transformation are found to have no significant role. Aside from the scale factor, this transformation is similar to that between WGS72 and WGS84.

To refine this transformation further will require more precise position estimates in SGS85 than are currently available, and must await further development of the GLONASS system and the availability of GLONASS receivers.



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