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# Due Regard Encounter Model Version 1.0

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This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

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# Version 1.0

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## ABSTRACT

Airspace encounter models describe encounter situations that may occur between aircraft in the airspace and are a critical component of safety assessment of sense and avoid (SAA) systems for Unmanned Aircraft Systems (UASs). Some UAS will fly in international airspace under *due regard* and may encounter other aircraft during these operations. In these types of encounters, the intruder aircraft is likely receiving air traffic control (ATC) services, but the UAS is not. Thus, there is a need for a due regard encounter model that can be used to generate these types of encounters. This report describes the development of a due regard encounter model. In order to build the model, Lincoln Laboratory collected data for aircraft flying in international airspace using the Enhanced Traffic Management System (ETMS) data feed that was provided by the Volpe Center. Lincoln processed these data, and extracted important features to construct the model. The model is based on Bayesian networks that represent the probabilistic relationship between variables that describe how aircraft behave. The model is used to construct random aircraft trajectories that are statistically similar to those observed in the airspace. A large collection of encounters generated from an airspace encounter model can be used to evaluate the performance of a SAA system against encounter situations representative of those expected to actually occur in the airspace.

Lincoln Laboratory has previously developed several other encounter models. There is an uncorrelated encounter model that is used to generate encounters with an intruder that does not have a transponder, or between two aircraft using a Mode A code of 1200 (VFR). There is also a correlated encounter model that is used when both aircraft have a transponder and at least one aircraft is in contact with ATC. Both of these models were built from radar data collected from the National Airspace System (NAS). There is also an unconventional encounter model that is used to generate encounters with unconventional intruders such as gliders, balloons, and airships—these vehicles have different flight characteristics than conventional aircraft. The framework used to construct the due regard encounter model described in this paper is similar to the prior models. The primary difference is that a different data feed is used and the model covers encounters in international flight where the aircraft of interest is flying due regard, which were not within the scope of prior models.

Separate electronic files are available from Lincoln Laboratory that contain the statistical data required to generate encounter trajectories.

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# 1. INTRODUCTION

Airborne safety critical systems must undergo extensive validation under realistic conditions prior to certification in the National Airspace System (NAS). Collision avoidance systems for manned aircraft and sense and avoid (SAA) systems for Unmanned Aircraft Systems (UASs) provide a safety critical function. They ensure separation when other safety layers have failed to maintain separation (e.g., the underlying airspace structure, airspace procedures, and air traffic control). These complex systems are assessed in realistic large scale Monte Carlo simulations and flight tests to prove that they meet the desired levels of safety. Fundamental to these simulations is the use of realistic encounter situations between aircraft which are defined by the relative geometry and behavior of the aircraft during the encounter. The geometry and dynamic behavior of aircraft during encounters are captured in encounter models. Encounter models provide a statistically sufficient set of features which are estimated from a large collection of observed encounter events. The initial and continuing evaluation of the Traffic Alert and Collision Avoidance System (TCAS) for manned aircraft illustrates the necessity of encounter models to estimate system effectiveness in a wide variety of encounter geometries [1–3].

Lincoln Laboratory has previously developed encounter models for the NAS. These models were built from radar data collected from radars across the NAS [4]. There are fundamentally two types of encounters: correlated and uncorrelated. In the first type, at least one aircraft in the encounter is receiving air traffic control (ATC) services, and both aircraft are transponder equipped. In the second type, ATC services would no longer be provided because either one aircraft is not transponder equipped or both aircraft are flying under visual flight rules (VFR). The first type is termed "correlated" because there is active coordination provided by ATC prior to the loss of separation. It is therefore critical that the model capture this coordination between aircraft prior to the loss of separation. In the second type of encounter, a lack of ATC services results in uncoordinated loss of separation. Hence, these encounters are termed "uncorrelated"—i.e., aircraft blunder into one another. This uncorrelated feature is exploited by modeling each aircraft individually and then simulating the uncoordinated loss of separation.

This report describes the development of an encounter model that can be used to evaluate SAA systems operating on UAS flying due regard in oceanic airspace. An aircraft flying *due regard* is not operating under International Civil Aviation Organization (ICAO) flight procedures and is responsible for maintaining its own separation from other aircraft [5]. More specifically, a due regard encounter model describes the statistical distribution of close encounter situations that are expected to occur in oceanic airspace when an UAS is operating due regard. A due regard encounter model is an uncorrelated model because ATC intervention is unlikely. The next section gives an overview of all the encounter models that Lincoln has developed and describes when it is appropriate to use each one. The following section then gives an overview of this report.

## 1.1 MODEL SELECTION

UASs are envisioned to operate in a variety of environments. As an example, the Broad Area Maritime Surveillance (BAMS) UAS will operate over the contiguous United States, in offshore environments, and in oceanic environments [6]. Lincoln has developed a variety of encounter models to evaluate SAA systems as a result of the wide range of operating environments for UASs. There are four types of encounter models:

- **Uncorrelated Encounter Model of the National Airspace System (U):** An uncorrelated encounter model is used to evaluate the performance of SAA systems when at least one aircraft is noncooperative or neither aircraft are in contact with ATC [7]. This model was developed from 1200-code aircraft observed in the NAS and this model now includes the offshore environment out to the limits of radar coverage [8].

- **Correlated Encounter Model of the National Airspace System (C):** A correlated encounter model is used to evaluate the performance of SAA systems when both aircraft are cooperative and at least one aircraft is receiving ATC services [9]. This model was developed from observed encounters in the NAS and also includes the offshore environment.

- **Encounter Models for Unconventional Aircraft (X):** This encounter model is used to evaluate the performance of an SAA system when encountering unconventional aircraft, defined as aircraft unlikely to carry a transponder [10]. Examples of unconventontial aircraft include balloons, blimps, gliders, and skydivers. This model was developed from GPS-recorded tracks that are posted online.

- **Due Regard Encounter Model (D):** This encounter model is used to evaluate SAA systems operating on UAS flying due regard in oceanic airspace. This model is primarily built from self-reported positions of aircraft flying in oceanic airspace.

Each of these encounter models includes variables that account for variations in encounters with respect to different airspaces. For example, one of the variables in the uncorrelated encounter model is Airspace Class, which includes B, C, D, and O (Other). Table 1 indicates the appropriate model to use based on the study being performed.[1] For the offshore environment, the correlated and uncorrelated encounter models encompass encounters more than 1 NM beyond the shore and the due regard model begins at 12 NM, where due regard flight is permitted.[2] The oceanic environment includes international airspace beyond radar coverage. Note that no existing model covers encounters between two IFR aircraft in oceanic airspace. The reason for this is that one cannot observe encounters of sufficient fidelity in the data feeds. Similarly, there is no model that covers encounters with VFR or noncooperative aircraft in oceanic airspace due to a lack of surveillance data. If a collection of encounters for these types of encounters is required, they should be built based on best assumptions about aircraft behavior and should leverage data from similar encounter models as is necessary. For example, one could use offshore models or enroute model of the Continental United States (CONUS).

[1] Note that a model does not exist for combinations without a mark.

[2] Note that one cannot build a *correlated* encounter model from radar data for due regard flight in the offshore environment because one does not observe a sufficient number of encounters between instrument flight rules (IFR) and non-IFR traffic beyond 12 NM.

**TABLE 1**

**Encounter model categories.**

| Aircraft of Interest | | Intruder Aircraft | | | |
|---|---|---|---|---|---|
| Location | Flight Rule | IFR | VFR | Noncooperative Conventional | Noncooperative Unconventional |
| CONUS | IFR | **C** | **C** | **U** | **X** |
| | VFR | **C** | **U** | **U** | **X** |
| Offshore | IFR | **C** | **C** | **U** | **X** |
| | VFR | **C** | **U** | **U** | **X** |
| | Due Regard | $\boxed{\textbf{D}}$ | **U** | **U** | **X** |
| Oceanic | IFR | | | | |
| | VFR | | | | |
| | Due Regard | $\boxed{\textbf{D}}$ | | | |

## 1.2 OVERVIEW

This document describes the due regard encounter model. Section 2 describes the Enhanced Traffic Management System (ETMS) data used to build the model. Section 3 describes the model itself. Section 4 describes how to process the ETMS data to build the due regard encounter model. Section 5 discusses some aspects of safety evaluation using the encounter model, which includes the process for initializing encounters and calculating the probability of near mid-air collision (NMAC) given an encounter. Lastly, Section 6 summarizes the model.

Note that the overall development process for this model closely follows that for the uncorrelated encounter model for the NAS. As is such, some development details and background are omitted from this report. The reader should refer to the uncorrelated model technical report for more information [7].

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# 2. DATA

ETMS is used by the Federal Aviation Administration (FAA) to support the strategic flow of aircraft operating in terrestrial and oceanic airspace. ETMS processes data from a variety of sources that include oceanic position updates, NAS messages, weather data, airline schedules, traffic data, and others. This information is then disseminated to support various traffic management functions such as traffic display, congestion predictions, congestion management, rerouting, and collaborative decision making [11].

Lincoln Laboratory maintains an ETMS data feed (now the Traffic Flow Management System) and uses a subset of the track data over oceanic airspace available in the feed to build the due regard encounter model. Track data is updated once a minute in ETMS and includes reported position, altitude, ground speed, and heading. Construction of the model used eight weeks during 2009 which were distributed throughout the year to prevent model bias due to seasonal variations. The collection periods were:

- 1 February 2009 through 14 February 2009,
- 3 May 2009 through 16 May 2009,
- 2 August 2009 through 14 August 2009, and
- 9 November 2009 through 22 November 2009.

After extracting tracks in the oceanic environment, 10,579 hours of track data were used to build the due regard encounter model. Figure 1 shows the average number of aircraft observed in the ETMS data within half degree by half degree bins across international airspace during the collection period. Only data for cells within one of the geographic domains for the due regard encounter model are presented. Cells with no data are white.

Previous encounter models have been built from radar data with five or twelve s update rates. Due to the fact that ETMS tracks are updated once every 60 s, it was necessary to first validate that ETMS data can be used to build an encounter model. This validation is described in Appendix B. The high-level approach involves building a site-specific encounter model from both radar and ETMS data from the same geographic region that includes many aircraft tracks in the North Atlantic Tracks (NAT). The models are built and compared to show that they are statistically similar.

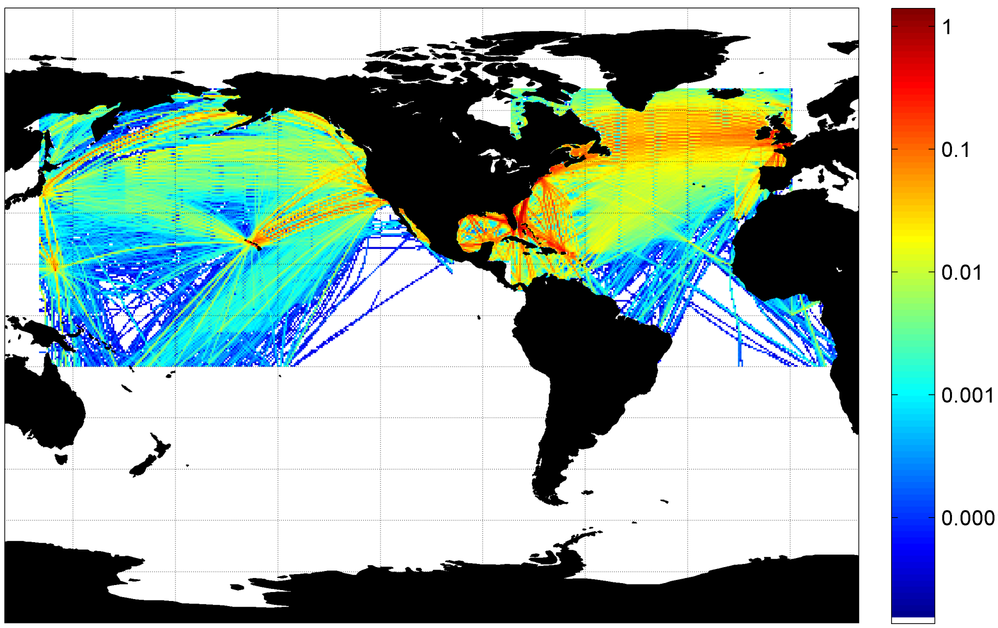


*Figure 1. Average number of aircraft present in half degree by half degree bins during data collection period.*

# 3. MODEL

Nominal flight, i.e., flight without avoidance maneuvering, is modeled using a Markov process represented by a dynamic Bayesian network. A Markov process is a stochastic process where the probability distribution over future states is conditionally independent of past states given the present state. In other words, one only needs to know the present state to predict the next state.

The states in the model specify how the position, altitude, and airspeed change over time. In particular, each state specifies a vertical rate $\dot{h}$, turn rate $\dot{\psi}$, and airspeed acceleration $\dot{v}$. Given an initial airspeed $v$, position, heading $\psi$, vertical rate $\dot{h}$, altitude layer $L$, and geographic domain $G$, one can infer from the model how the aircraft trajectory evolves over time.

One way to represent a Markov model is with an exhaustive state-transition matrix that specifies the probability of transitioning between all pairs of states. Unfortunately, the number of independent parameters required to define the matrix grows super exponentially with the number of variables defining the model. The more independent parameters that are in the model, the more data one needs to properly estimate their values. However, by using dynamic Bayesian networks, one can leverage conditional independence between some variables to greatly reduce the number of parameters. One can learn the structure of the dynamic Bayesian network by maximizing the posterior probability of the network structure given the data [4].

The Bayesian networks that maximized the posterior probability of the structure can be determined by searching the space of network structures. The general process involves guessing a fully connected network structure and scoring that structure. A collection of networks is then constructed where for each network a single conditional dependency is removed between two variables in the network and score them. If any of the new networks has a higher probability, it is kept and the search continues. The search terminates once removing any conditional dependency in the network decreases the probability.

## 3.1 MODEL VARIABLES

There are seven variables in the due regard encounter model:

- **Altitude layer $L$:** Airspace is divided into six altitude layers, in a process similar to prior encounter models. The first layer spans up to 5500 ft Mean Sea Level (MSL). The second layer spans 5500 to 10,000 ft MSL. The third layer spans from 10,000 ft MSL to FL180. The fourth through sixth layers are FL180 to FL290, FL290 to FL410, and above FL410.
- **Geographic Domain $G$:** The due regard encounter model is separated into domains to capture variations in aircraft behavior as a result of geographic area. This is a new variable for encounter modeling, which is discussed in more detail below.
- **Airspeed $v$:** True airspeed is modeled and allowed to vary during flight. Units are in kts.
- **Heading $\psi$:** Aircraft heading is modeled in the due regard encounter model and it was allowed to vary during flight. This variable is unique to the due regard encounter model

because of the unique, regular route structures in much of international airspace. Units are in degrees

- **Acceleration $\dot{v}$:** Airspeed acceleration is allowed to vary every second. Units are in kts/s.
- **Turn rate $\dot{\psi}$:** Turn rate is permitted to change every second in the model. Units are degrees/s.
- **Vertical rate $\dot{h}$:** The vertical rate is permitted to change at every second. Units are ft/min.

Because many of the variables are closely related due to physical constraints and flight characteristics (e.g., turn rate and vertical rate), it is important to properly represent correlations in the model. Independently sampling from distributions for turn rate and vertical rate would ignore these important relationships. The remainder of this section explains how to model joint probability distributions over these variables to ensure proper consideration of correlations. To generate an encounter, one first randomly samples from the joint distribution over the encounter variables to define the initial conditions. A Markov model is then used to determine how the dynamic variables (i.e., turn rate, vertical rate, and airspeed acceleration) evolve during the course of the encounter. There are two corresponding separate probability distributions in the model: an initial distribution to set up an encounter situation and a transition distribution to describe how the dynamic variables specifying the trajectories of the aircraft evolve over time.

The geographic domains in the due regard encounter model are listed in Table 2 and are depicted in Figure 2. Table 2 also shows the longitudinal and latitudinal limits for the geographic domains in the due regard encounter model. Note that the limits only apply in international airspace—e.g., airspace greater than 12 NM from land. Thus, aircraft in one of these domains but over land are not part of the model. The upper longitudinal limit (66°33′44″ N) corresponds to the arctic circle. These domains were selected because they encompass U.S.-controlled oceanic airspace and they broadly capture areas with repeatable and similar tracks—e.g., tracks with similar magnetic headings.

**TABLE 2**

**Geographic domain limits.**

| | | Longitudinal Limits | Latitudinal Limits |
|---|---|---|---|
| 1 | North Pacific | 37°0′0″ N to 66°33′44″ N | 140°0′0″ E to 98°0′0″ W |
| 2 | West Pacific | 15°0′0″ S to 37°0′0″ N | 140°0′0″ E to 158°0′0″ W |
| 3 | East Pacific | 15°0′0″ S to 37°0′0″ N | 158°0′0″ W to 99°0′0″ W |
| 4 | Gulf of Mexico | 16°0′0″ N to 30°48′ N | 98°0′0″ W to 82°0′0″ W |
| 5 | Caribbean | 7°30′0″ N to 37°0′0″ N | 82°0′0″ W to 60°0′0″ W |
| 6 | North Atlantic | 37°0′0″ N to 66°33′44″ N | 82°0′0″ W to 0°0′0″ |
| 7 | Central Atlantic | 15°0′0″ S to 37°0′0″ N | 60°0′0″ W to 15°0′0″ E |

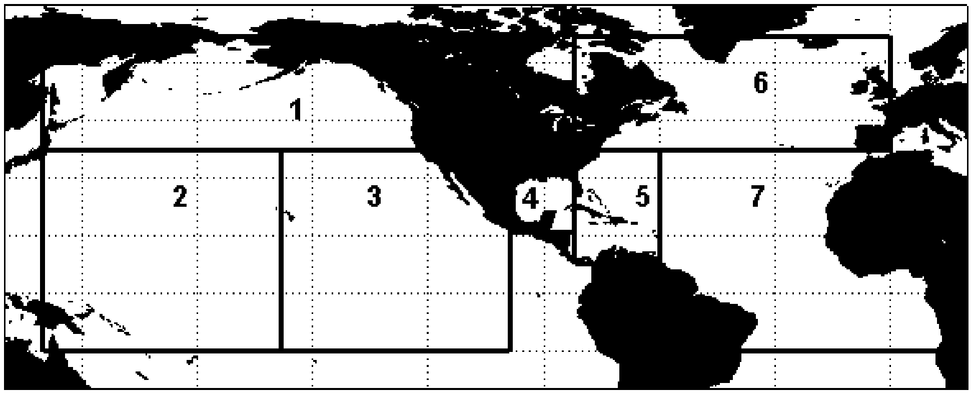


*Figure 2. Geographic domains.*

### 3.2 INITIAL DISTRIBUTION

It is important that the model of aircraft flight captures the distribution over the initial values of $\dot{h}$, $\dot{\psi}$, $\dot{v}$, $\psi$, $v$, $L$, and $G$. Figure 3 shows the structure used for the modeling effort. Other structures are certainly plausible, and it is possible to compare different structures using Bayesian scoring to determine which structure is more likely given the data [4]. The network structure in Figure 3 was chosen because it was the most likely among the selection of candidate networks considered in the structure search. Appendix A contains figures showing marginal distributions for the individual variables in the model.

Given a structure, sufficient statistics extracted from data, and a Bayesian prior, one can sample from the Bayesian network to produce sets of initial airspace classes, altitude layers, vertical rates, turn rates, airspeeds, and accelerations that are representative of those found in the data. The boxes and arrows used in the structural diagram show the order in which this sampling occurs. For example, based on the structure in Figure 3, to determine the initial state of the aircraft one would first randomly determine an altitude layer $L$. Once the altitude layer has been determined, a geographic domain $G$ is selected. The probabilities associated with each geographic domain depend on which altitude layer was chosen earlier. Once $L$ and $G$ have been selected, the model randomly selects airspeed $v$, and so on. Alternately, one could assign outright a geographic domain and/or altitude layer for a particular study and then randomly select values for the remaining variables.

### 3.3 TRANSITION DISTRIBUTION

A separate Bayesian network is used to model how the variables $\dot{h}$, $\dot{\psi}$, and $\dot{v}$ evolve over time (see Figure 4). In this network, the first layer represents the state of the trajectory at the present

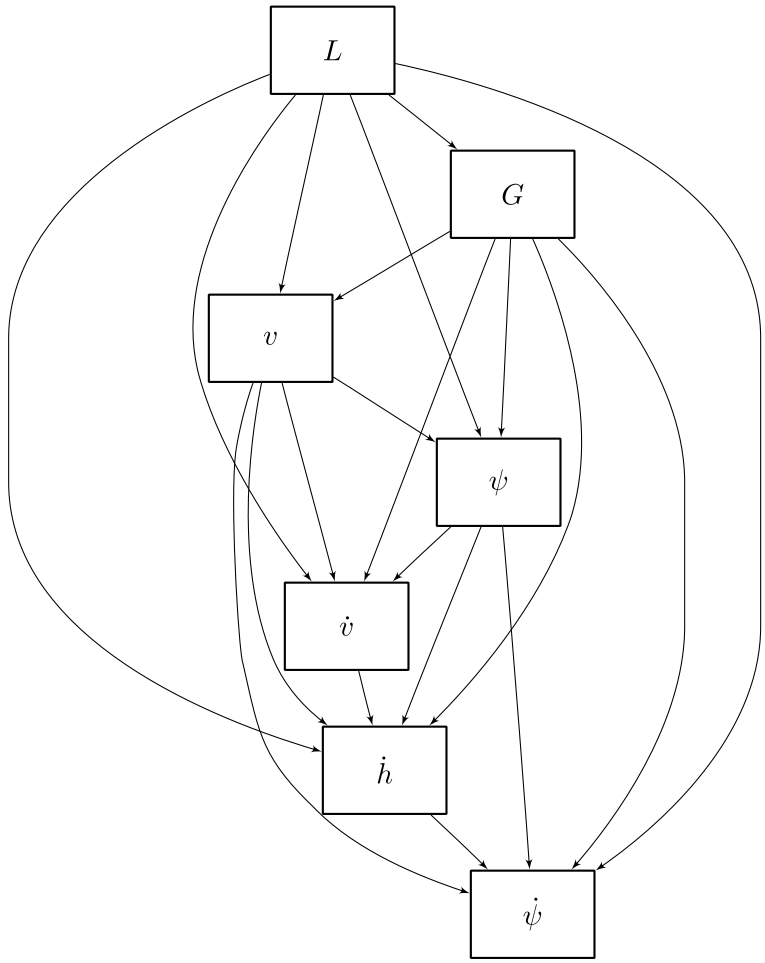


*Figure 3. Initial structure.*

time step and a second layer that represents the state of the trajectory at the next time step. There may be dependencies between layers and within the second layer. Such a two-layer temporal Bayesian network is known as a dynamic Bayesian network [12]. Parameter and structure learning in dynamic Bayesian networks is similar to regular Bayesian networks. Again, the highest-scoring network structure was chosen among the candidate network structures. Given a structure, sufficient statistics extracted from data, and a prior, one then samples from the Bayesian network to determine the next vertical rate, turn rate, and acceleration command that are representative of what was observed in the data.

In general, time steps in dynamic Bayesian networks may be of any duration, but for the encounter modeling effort a time step of 1 s was used. Shorter time steps allow for more frequent variations in airspeed, vertical rate, and turn rate, but they require more computation per unit of simulation time. Time steps of 1 s balance maneuver complexity with computation.

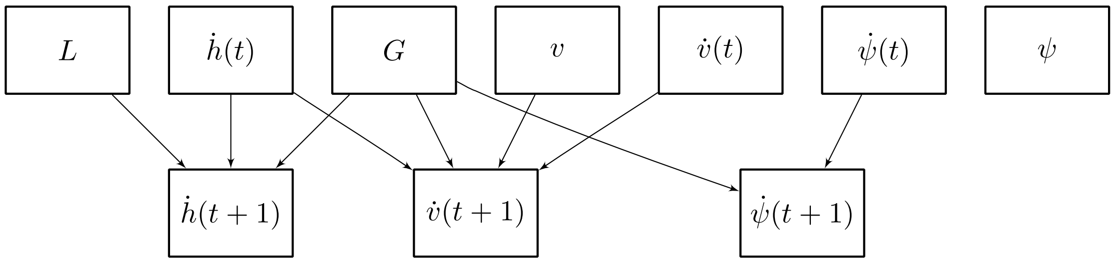


*Figure 4. Transition structure.*

A complete trajectory is constructed by updating the aircraft state in 1 s intervals. Within each interval, the three derivative variables $\dot{h}$, $\dot{\psi}$, and $\dot{v}$ are treated as target values and held constant. A dynamic model (which is beyond the scope of this report) is used to compute and update the aircraft state at each time step based on these piecewise-constant target values. The encounter model can generate statistically representative trajectories that persist for more than 1000 s. In order to determine this, a statistical test of the encounter model was performed, which is described in Appendix C.

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# 4. PROCESSING

Figure 5 shows the processing flow to build the due regard encounter model. The first step was spatially filtering tracks to extract tracks in international airspace. The next step involves removing outliers. Next, the tracks are interpolated and the model features are extracted. The features are then quantized and extracted to estimate the sufficient statistics. The remainder of this section describes this process in more detail.

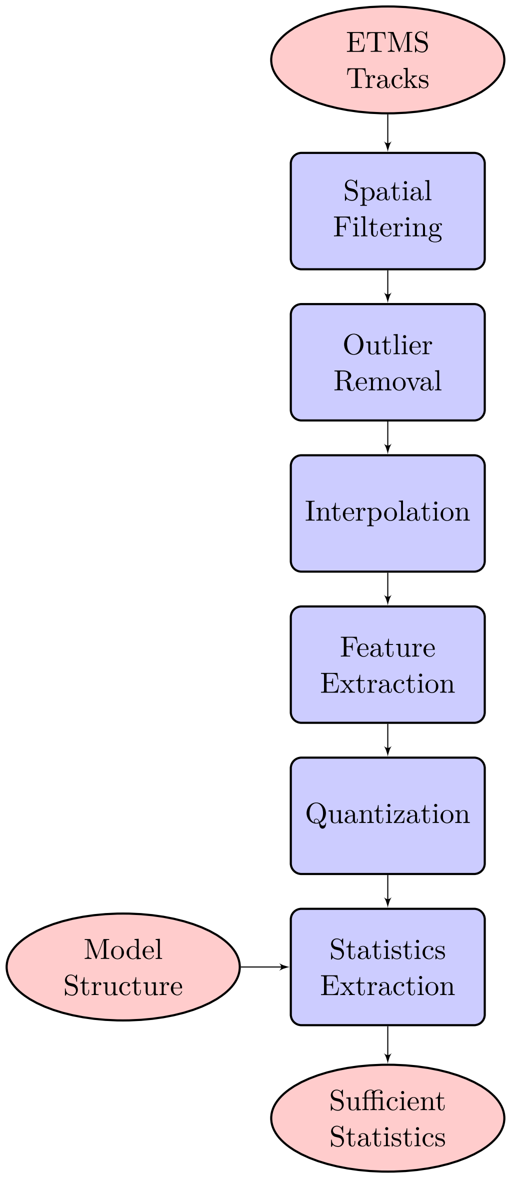


*Figure 5. Estimation process flow.*

### 4.1 OUTLIER REMOVAL

There are some outliers in the ETMS data. Thus, these outliers first have to be detected and removed so that the model accurately reflects aircraft motion. Outliers are detected in altitude, ground speed, and heading:

- **Altitude:** A reported altitude is assumed to be an outlier if the change in altitude is more than 10,000 ft over one minute. Some altitudes are reported as 0 ft—these are assumed to be outliers, as well.
- **Ground Speed:** A reported ground speed is assumed to be an outlier if the speed is less than 50 kts or greater than 650 kts. If the change in ground speed is greater than 120 kts between 60 s reports, then it is assumed to be an outlier, as well.
- **Heading:** A reported heading is assumed to be an outlier if a heading is reported as 0 or 360 degrees *and* the change in heading is more than 60 degrees between the previous or next report. For example, if a sequence of reported headings were $88, 88, 87, 0, 87, 87$, then the 0 is removed.

Processing also compares the reported ground speed and heading against values that are calculated from consecutive position reports. If the calculated ground speed difference is greater than 100 kts, then an outlier is assumed. If the calculated heading difference is greater than 25 degrees, then an outlier is also assumed.

### 4.2 INTERPOLATION

Next, the altitude, ground speed, and headings are interpolated. Interpolation is done at 1 s using a shape preserving piecewise-cubic interpolation. Note that interpolation does not occur between reports if there is a missing report between them—i.e., the time between reports is greater than 60 s.

### 4.3 FEATURE EXTRACTION

Recall that the variables in the model are $L$, $G$, $v$, $\psi$, $\dot{v}$, $\dot{\psi}$, and $\dot{h}$. The variable $L$ is extracted directly from the altitude reports. The variable $G$ is estimated using the nearest reported latitude and longitude. Airspeed is assumed to be equivalent to ground speed. The variable $\psi$ is extracted directly from heading. Finally, $\dot{v}$, $\dot{\psi}$, and $\dot{h}$ are extracted by calculating the derivative of ground speed, heading, and altitude, respectively.

### 4.4 QUANTIZATION

In order to be modeled by a discrete Bayesian network, it is necessary to quantize the features. Continuous values are quantized by defining a sequence of cut points $c_1, \ldots, c_n$. Values less than

$c_1$ are in the first bin, values greater than $c_n$ are in the $(n+1)$th bin, and values in the half-open interval $[c_{i-1}, c_i)$ are in the $i$th bin. The cut points used for quantization are listed in Table 3.

**TABLE 3**

**Cut points used for feature quantization.**

| | Cut Points |
|---|---|
| $L$ | 5500 ft, 10000 ft, FL180, FL290, FL410 |
| $G$ | 1, 2, 3, 4, 5, 6, 7 |
| $v$ | 200, 300, 400, 500 |
| $\psi$ | 60, 120, 180, 240, 300 |
| $\dot{v}$ | -1.5, -0.25, 0.25, 1.5 |
| $\dot{h}$ | -3000, -2000, -1000, -400, 400 1000, 2000, 3000 |
| $\dot{\psi}$ | -3.5, -1, -0.25, 0.25, 1, 3.5 |

## 4.5 STATISTICS EXTRACTION

With structures for the initial and transition distributions and the quantized features from a set of tracks, the sufficient statistics can be collected to estimate the parameters for the model. For the two Bayesian networks, the sufficient statistics are simply the counts of the various features (see Appendix I in [7]).[3] Appendix I in [7] describes the sufficient statistics and Bayesian networks in greater detail.

[3]The counts are called *sufficient statistics* because they provide a summarization of the data that is sufficient to compute the posterior distribution from the prior. For an introduction to Bayesian statistics, see [13].

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# 5. SAMPLING, SIMULATION, AND SAFETY EVALUATION

Once the data have been processed as described in the previous section, one can use the model to produce new trajectories that are representative of those observed in the ETMS data by sampling from the Bayesian networks. This process is carefully described in Section 4 of [7]. The next step is to initialize the trajectory of an intruder aircraft on the encounter cylinder centered about the aircraft of interest. Section 5 of [7] describes this process. However, there are some important changes in this model. Thus, Section 5.1 of this document describes the process to initialize encounter situations that should be used with this model. Section 6 of [7] then described how to estimate NMAC probability. Due to the fact that a different initialization process is used in this model, the appropriate procedure for estimating NMAC probability is different. It is discussed in Section 5.2 of this document.

## 5.1 ENCOUNTER INITIALIZATION

Rejection sampling is used to generate the initial conditions of an encounter. Rejection sampling involves proposing a series of candidate samples from a random distribution until choosing one that meets a set of criteria. The process used for generating initial conditions for encounters is as follows:

1. Generate a set of initial conditions for the due regard aircraft (AC1). The initial conditions include altitude layer, geographic domain, airspeed, heading, vertical rate, turn rate, and acceleration.
2. Sample from the Bayesian network to generate AC2. If the sample has the same altitude layer and geographic domain, then keep AC2; otherwise, reject the sample and continue to sample until the altitude layer and geographic domain match AC1.[4] AC1 and AC2 are termed the own and intruder aircraft, respectively.
3. Calculate the velocity vectors for both aircraft ($\mathbf{v}_1$ and $\mathbf{v}_2$) based on the aircraft initial conditions. The relative velocity vector $\mathbf{v}_\mathrm{r}$ is $\mathbf{v}_1$ subtracted from $\mathbf{v_2}$. See Figure 6.
4. Determine the projected surface $\mathcal{S}$ of the cylinder onto a plane that is perpendicular to $\mathbf{v}_\mathrm{r}$.
5. The intruder aircraft penetrates the encounter cylinder uniformly over $\mathcal{S}$. Therefore, uniformly select a random point $p$ inside $\mathcal{S}$.
6. Project this point back onto the encounter cylinder. There will be two candidate points. Select the one such that the intruder aircraft is penetrating the encounter cylinder. The following tests can be used to determine which candidate initial point is correct:
   - If AC2 was initialized on the top of the encounter cylinder, accept the sample if the vertical rate of AC2 relative to AC1, denoted $\mathbf{v}_{\mathrm{r,v}}$, is negative. This ensures that AC2 is penetrating the encounter cylinder for the first time.

[4]Note that this will result in an incorrect distribution over layer and geographic domain (compared to that observed). Section 5.3 describes how to correct for this.

- If AC2 was initialized on the bottom of the encounter cylinder, accept the sample if the vertical rate of AC2 relative to AC1, denoted $\mathbf{v}_{\mathrm{r,v}}$, is positive. This ensures that AC2 is penetrating the encounter cylinder for the first time.
- If AC2 was initialized on the side of the encounter cylinder, accept the sample if $\hat{\mathbf{R}}_{\mathrm{h}} \cdot \mathbf{v}_{\mathrm{r,h}}$ is negative. Here $\hat{\mathbf{R}}_{\mathrm{h}}$ is the horizontal component of $\hat{\mathbf{R}}$, which is the relative position of AC2 from AC1, and $\mathbf{v}_{\mathrm{r,h}}$ is $\mathbf{v}_{1,\mathrm{h}}$ subtracted from $\mathbf{v}_{2,\mathrm{h}}$. The vectors $\mathbf{v}_{1,\mathrm{h}}$ and $\mathbf{v}_{2,\mathrm{h}}$ are the horizontal velocities of AC1 and AC2 respectively. If $\hat{\mathbf{R}}_{\mathrm{h}} \cdot \mathbf{v}_{\mathrm{r,h}}$ is negative, then accept the encounter because AC2 is penetrating the cylinder about AC1.

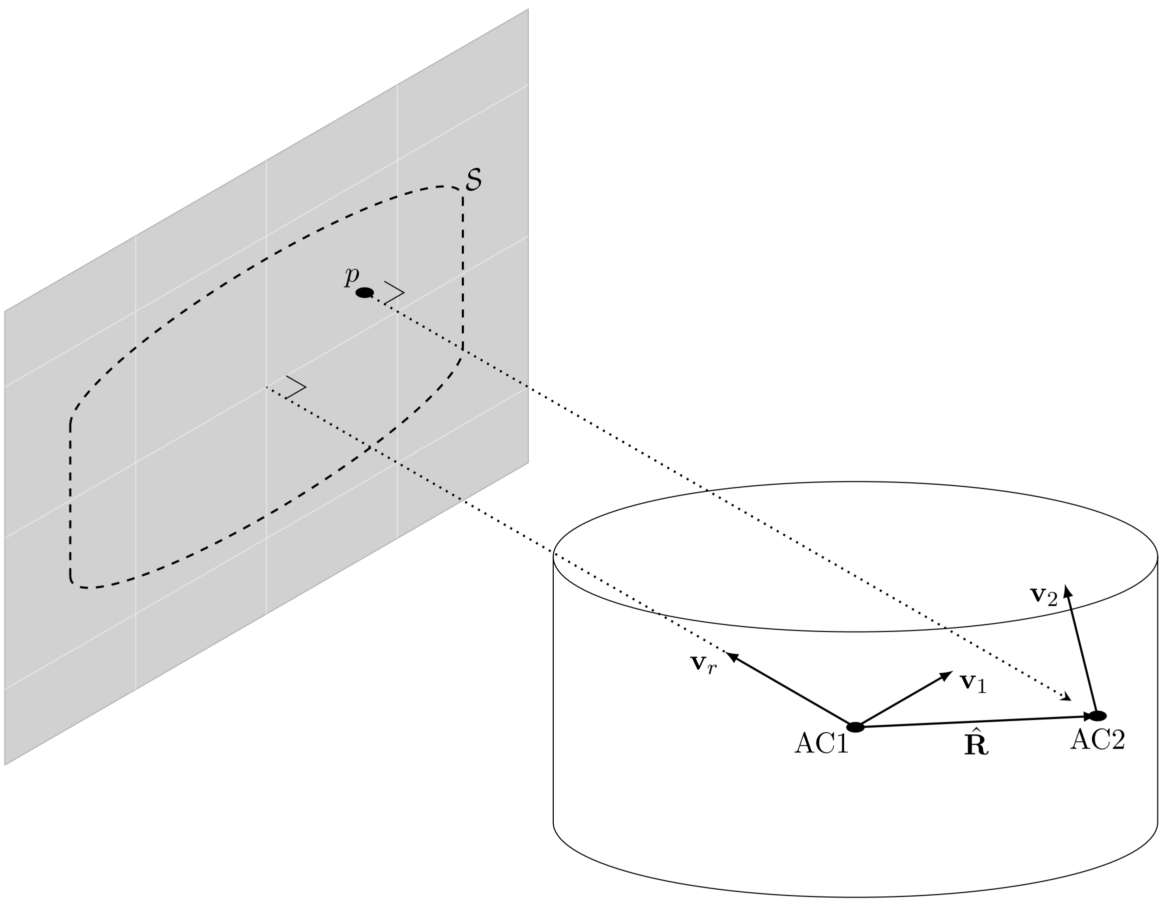


*Figure 6. Initialization process.*

### 5.2 ESTIMATING NMAC PROBABILITY

Section 5.1 explained how to construct an encounter from two independent trajectories sampled from the distribution represented by Bayesian networks. By generating a large collection of encounters and determining which encounters lead to NMACs, one can estimate $P(\text{nmac} \mid \text{enc})$. Unfortunately, one cannot simply divide the number of sampled encounters that lead to NMACs by the total number of sampled encounters to estimate $P(\text{nmac} \mid \text{enc})$ due to the fact that the sampling scheme does not produce encounters from the same distribution that would occur in the airspace. In particular, the model generates encounters with aircraft velocities distributed identically to the aircraft population at large, despite the fact that in reality the distribution of aircraft velocities given that an encounter is occurring favors high-speed aircraft. Although one samples from a distribution that is different from the true distribution when constructing encounters, one can still use the samples to estimate $P(\text{nmac} \mid \text{enc})$ so long as one weights their results properly using an approach known as importance sampling [14]. This section begins by stating the weighting scheme and then proves that it is correct.

Section 4 of [7] explains how to generate the trajectories for AC1 and AC2, $\mathbf{z}_1$ and $\mathbf{z}_2$, by sampling from the Bayesian networks with the requirement that both aircraft come from the same altitude layer and geographic domain. Section 5.1 then explained how to randomly select the position and orientation of AC2 relative to AC1, which is termed $\mathbf{x}_r$. Importance sampling allows one to make the following approximation based on $N$ samples

$$P(\text{nmac} \mid \text{enc}) \approx \frac{1}{N} \sum_i P(\text{nmac} \mid \mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)}, \mathbf{x}_r^{(i)}, \text{enc}) \frac{V(\mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)})}{\bar{V}} .$$

The weight $V(\mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)})/\bar{V}$ corrects for the fact that the sampling distribution does not match the true distribution of encounter situations. The function $V(\mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)})$ is the average volume the encounter cylinder sweeps out per unit time when AC1 follows $\mathbf{z}_1^{(i)}$ and the airspace consists exclusively of aircraft following $\mathbf{z}_2^{(i)}$. In particular,

$$V(\mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)}) = |\mathbf{v_r}|\, a(\mathcal{S}) ,$$

where $|v_r|$ is the magnitude of the relative velocity vectors and $a(\mathcal{S})$ is the area of $\mathcal{S}$. The constant $\bar{V}$ is the average volume the encounter cylinder sweeps out per unit time

$$\bar{V} = \iint p(\mathbf{z}_1) p(\mathbf{z}_2 \mid \mathbf{z}_1) V(\mathbf{z}_1, \mathbf{z}_2)\, \mathrm{d}\mathbf{z}_1 \mathrm{d}\mathbf{z}_2 .$$

Note that the distribution over $\mathbf{z}_2$ is conditional on $\mathbf{z}_1$ due to the constraint that AC1 and AC2 must belong to the same airspace class and altitude layer. The constant $\bar{V}$ can be estimated using $N$ samples:

$$\bar{V} \approx \frac{1}{N} \sum_i V(\mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)}) . \tag{1}$$

Now that the weighting scheme has been defined, this section now proves that it is correct. From the laws of probability,

$$\begin{aligned} P(\text{nmac} \mid \text{enc}) &= \iint P(\text{nmac} \mid \mathbf{z}_1, \mathbf{z}_2, \text{enc}) p(\mathbf{z}_1, \mathbf{z}_2 \mid \text{enc})\, \mathrm{d}\mathbf{z}_1 \mathrm{d}\mathbf{z}_2 \\ &= \iiint P(\text{nmac} \mid \mathbf{z}_1, \mathbf{z}_2, \mathbf{x}_r, \text{enc}) p(\mathbf{x}_r \mid \mathbf{z}_1, \mathbf{z}_2, \text{enc}) p(\mathbf{z}_1, \mathbf{z}_2 \mid \text{enc})\, \mathrm{d}\mathbf{z}_1 \mathrm{d}\mathbf{z}_2 \mathrm{d}\mathbf{x}_r . \end{aligned}$$

$P(\text{nmac} \mid \text{enc})$ may be approximated using Monte Carlo sampling. Since it is difficult to sample from $p(\mathbf{z}_1, \mathbf{z}_2 \mid \text{enc})$ directly, one can sample $\mathbf{z}_1$ and $\mathbf{z}_2$ from the distribution represented by the Bayesian network subject to the constraint that both aircraft come from the same geographic domain and altitude layer, and weight the samples appropriately:

$$P(\text{nmac} \mid \text{enc}) \approx \frac{1}{N} \sum_i P(\text{nmac} \mid \mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)}, \mathbf{x}_r^{(i)}, \text{enc}) \frac{p(\mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)} \mid \text{enc})}{p(\mathbf{z}_1^{(i)}) p(\mathbf{z}_2^{(i)} \mid \mathbf{z}_1^{(i)})} .$$

Next,

$$\begin{aligned} p(\mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)} \mid \text{enc}) &= \frac{p(\mathbf{z}_1^{(i)}) p(\mathbf{z}_2^{(i)} \mid \mathbf{z}_1^{(i)})}{\lambda_{\text{enc}}} \lambda_{\text{enc} \mid \mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)}} \\ &\propto p(\mathbf{z}_1^{(i)}) p(\mathbf{z}_2^{(i)} \mid \mathbf{z}_1^{(i)}) \lambda_{\text{enc} \mid \mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)}} \\ &\propto p(\mathbf{z}_1^{(i)}) p(\mathbf{z}_2^{(i)} \mid \mathbf{z}_1^{(i)}) V(\mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)}) . \end{aligned}$$

This result may be normalized to obtain

$$\begin{aligned} p(\mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)} \mid \text{enc}) &= p(\mathbf{z}_1^{(i)}) p(\mathbf{z}_2^{(i)} \mid \mathbf{z}_1^{(i)}) V(\mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)}) / \iint p(\mathbf{z}_1) p(\mathbf{z}_2 \mid \mathbf{z}_1) V(\mathbf{z}_1, \mathbf{z}_2) \, \mathrm{d}\mathbf{z}_1 \mathrm{d}\mathbf{z}_2 \\ &= p(\mathbf{z}_1^{(i)}) p(\mathbf{z}_2^{(i)} \mid \mathbf{z}_1^{(i)}) V(\mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)}) / \bar{V} . \end{aligned}$$

Substitution and simplification leads to

$$\begin{aligned} P(\text{nmac} \mid \text{enc}) &\approx \frac{1}{N} \sum_i P(\text{nmac} \mid \mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)}, \mathbf{x}_r^{(i)}, \text{enc}) \frac{p(\mathbf{z}_1^{(i)}) p(\mathbf{z}_2^{(i)} \mid \mathbf{z}_1^{(i)}) V(\mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)}) / \bar{V}}{p(\mathbf{z}_1^{(i)}) p(\mathbf{z}_2^{(i)} \mid \mathbf{z}_1^{(i)})} \\ &\approx \frac{1}{N} \sum_i P(\text{nmac} \mid \mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)}, \mathbf{x}_r^{(i)}, \text{enc}) \frac{V(\mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)})}{\bar{V}} , \end{aligned}$$

which corresponds to the weighting scheme defined above.

### 5.3 CORRECTING FOR LAYER AND GEOGRAPHIC DOMAIN

Encounters are generated across altitude layers and geographic domains according to their distributions defined in the model. These distributions represent the observed rate of occurrence of aircraft in each altitude layer and geographic domain. However, the expected encounter rate is proportional to the airspace density, *not* the cumulative occurrence of aircraft, in the local airspace. Furthermore, the aircraft of interest may be exposed to distribution of encounters in altitude layers and geographic domains that is different than the distributions in the model because the aircraft of interest has a greater exposure time ($t_e$) to encounters in other altitude layer and geographic domain combinations. Therefore, the mean NMAC probability over all geographic domains and altitude layers that considers these factors a posteriori is

$$\bar{P}(\text{nmac} \mid \text{enc}) = \sum_i P(\text{nmac} \mid \text{enc}, a_i, l_i) P(a_i, l_i \mid \text{enc}) = \frac{\sum_i P(\text{nmac} \mid \text{enc}, a_i, l_i) \rho_i \bar{V}_i t_e^{(i)}}{\sum_i \rho_i \bar{V}_i t_e^{(i)}} , \qquad (2)$$

where $i$ denotes each altitude layer and geographic domain combination. The term $P(l_i, g_i \mid \text{enc})$ is the proportion of encounters expected in each altitude layer and geographic domain combination.

If one knows $\rho_i$, $\bar{V}_i$, and $t_e^{(i)}$ a priori, then the altitude and airspace class distributions can be modified to reflect this knowledge before sampling from the model. Then, the mean NMAC probability estimate is simply the NMAC probability for all encounters. This sampling procedure may be useful when the aircraft of interest is expected to operate in a specific geographic domain or altitude layer. If little is known about the expected operating environment, then an objective (i.e., uniform) assumption regarding the geographic domain and altitude layer may be suitable.

## 5.4 ALTERNATIVE DISTRIBUTIONS

Some analyses may require that encounters be generated from an alternative distribution than that produced by the model using the procedure described earlier in this section. For example, one may expect a specific distribution over headings or horizontal miss distance (HMD) that is different than that generated by the model. This may be a result of a specific operation, for example. This section describes how to account for alternative distributions.

There are two relevant approaches. The first involves employing a procedure for manipulating the Bayesian network, which is only applicable in some circumstances. Another approach called *likelihood weighting* [15] involves weighting the samples to correct for the fact the sampled distribution does not match the expected distribution.

### 5.4.1 Bayesian Network Manipulation

If one samples a subset of the encounter variables in the Bayesian network $\mathcal{A} \subset \mathcal{X}$ from a distribution $P_{\text{alt}}(\mathcal{A})$ that is different from the encounter model distribution $P(\mathcal{X})$, special care must be taken when sampling the remaining variables so that the relationships in the Bayesian networks are preserved. In other words, after sampling the variables in $\mathcal{A}$ sampling then occurs over the remaining variables $\mathcal{B} = \mathcal{X} \setminus \mathcal{A}$ according to $P(\mathcal{B} \mid \mathcal{A})$ as inferred from the Bayesian network.

Simply assigning variables in $\mathcal{A}$ to values according to $P_{\text{alt}}(\mathcal{A})$ is permissible so long as the remaining variables $\mathcal{B}$ come later in a topological sort of $\mathcal{X}$.

Another way to ensure that the distribution of the remaining variables reflects the encounter model distribution is to use logic sampling [16]. First, sample the variables in $\mathcal{A}$ according to $P_{\text{alt}}(\mathcal{A})$. Then, sample from the encounter model Bayesian network until a sample is obtained that matches the bins that were assigned from the alternative distribution. The probabilistic logic sampling approach can require the generation of many samples before acceptance.

Alternately, one can manipulate the graphical structure of the encounter model Bayesian network to make the variables in $\mathcal{A}$ appear earlier than the variables in $\mathcal{B}$ in a topological sort. Then, the variables in $\mathcal{A}$ can be assigned directly according to $P_{\text{alt}}(\mathcal{A})$ and then the remaining variables $\mathcal{B}$ can be assigned using forward sampling. Special care must be taken when changing the direction of edges (also called arcs) in the Bayesian network to preserve the relationships between variables. If one wishes to reverse the direction of an edge between two nodes $X_i$ and $X_j$ in a Bayesian network, one must add the parents of $X_j$ to $X_i$ and vice versa. This *arc reversal* operation preserves the original distribution over $\mathcal{X}$ [17, 18]. A downside to arc reversal is that the resulting network can become very complex.

#### 5.4.2 Likelihood Weighting

Other analyses may require that encounters being generated from a distribution that cannot be accounted for by modifying the Bayesian network. For example, an analysis may require that the distribution over HMD be different than the distribution generated by the model. Note that HMD is not a variable in the Bayesian network, but is a product of sampling from the network and initializing the encounter. If this is the case, then each sample should be weighted according to the likelihood of a particular HMD value occurring in the airspace divided by the likelihood of generating that value of HMD using the model. More generally,

$$P(\text{nmac} \mid \text{enc}) \approx \frac{1}{N} \sum_i P(\text{nmac} \mid \mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)}, \mathbf{x}_r^{(i)}, \text{enc}) \frac{V(\mathbf{z}_1^{(i)}, \mathbf{z}_2^{(i)})}{\bar{V}} \frac{g(\mathbf{y}^{(i)})}{f(\mathbf{y}^{(i)})} ,$$

where $\mathbf{y}^{(i)}$ represents the value of the variable (or variables) under consideration for likelihood weighting, $g(\mathbf{y}^{(i)})$ is the density over $\mathbf{y}$ that occurs in the airspace, and $f(\mathbf{y}^{(i)})$ is the sampling density of $\mathbf{y}$ generated by the model.

# 6. SUMMARY

This document described the development of a due regard encounter model. This model should be used to evaluate SAA systems operating on UAS flying due regard in international airspace. This model assumes that aircraft blunder into close proximity without prior intervention. In this model the trajectories of the aircraft involved in the encounter are independent of each other prior to collision avoidance or SAA system intervention.

The process to develop this model closely follows the approach to develop other recent encounter models. The approach involved modeling aircraft dynamics as a Markov process where the state of the aircraft in the future probabilistically depends on its current state. The model is encoded in a dynamic Bayesian network, which efficiently captures the statistical dependencies between variables in the model.

The key difference between this model and others is the data used to build the model. Previous models have used radar data with 5 or 12 s updates. However, this model is based on ETMS data with a 60 s update. Part of this model development involved validating that ETMS data can be used to build a due regard encounter model.

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# APPENDIX A
# VARIABLE DISTRIBUTIONS

Figures A-1 through A-7 show marginal distributions for the variables in the due regard encounter model.

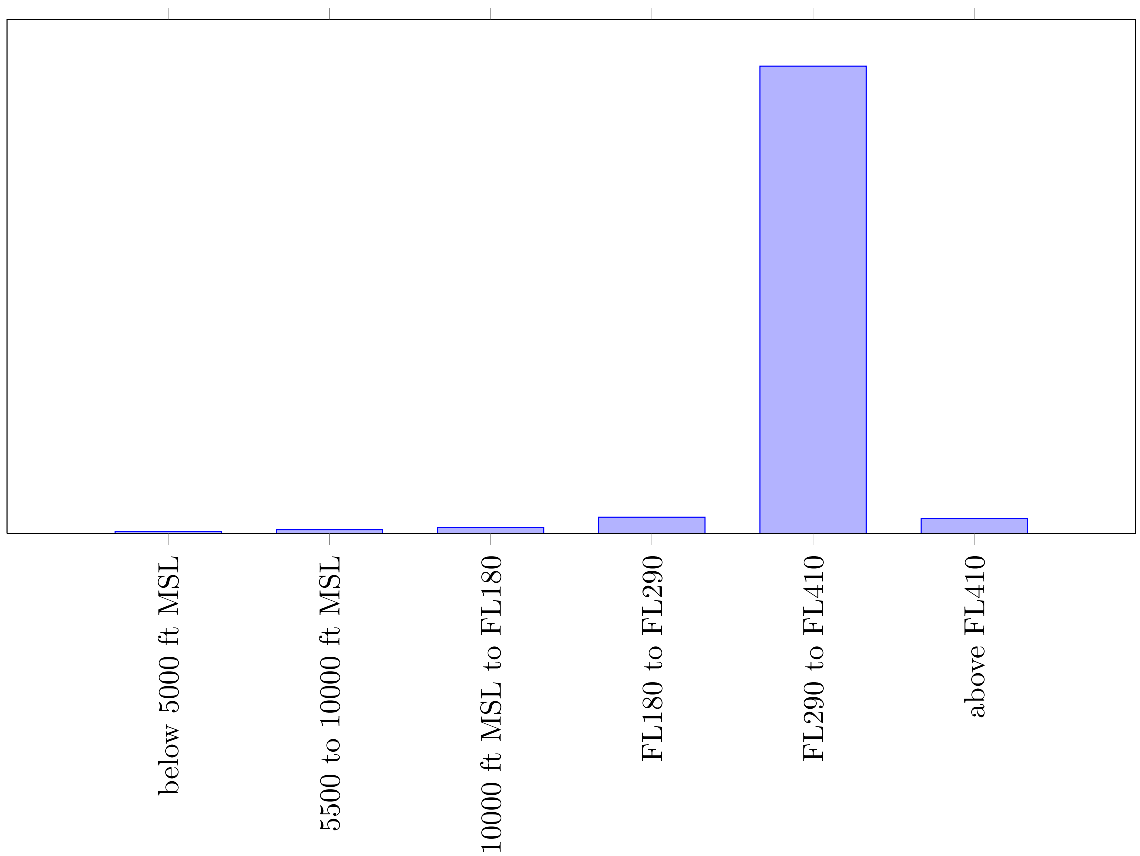


*Figure A-1. Altitude layer distribution.*

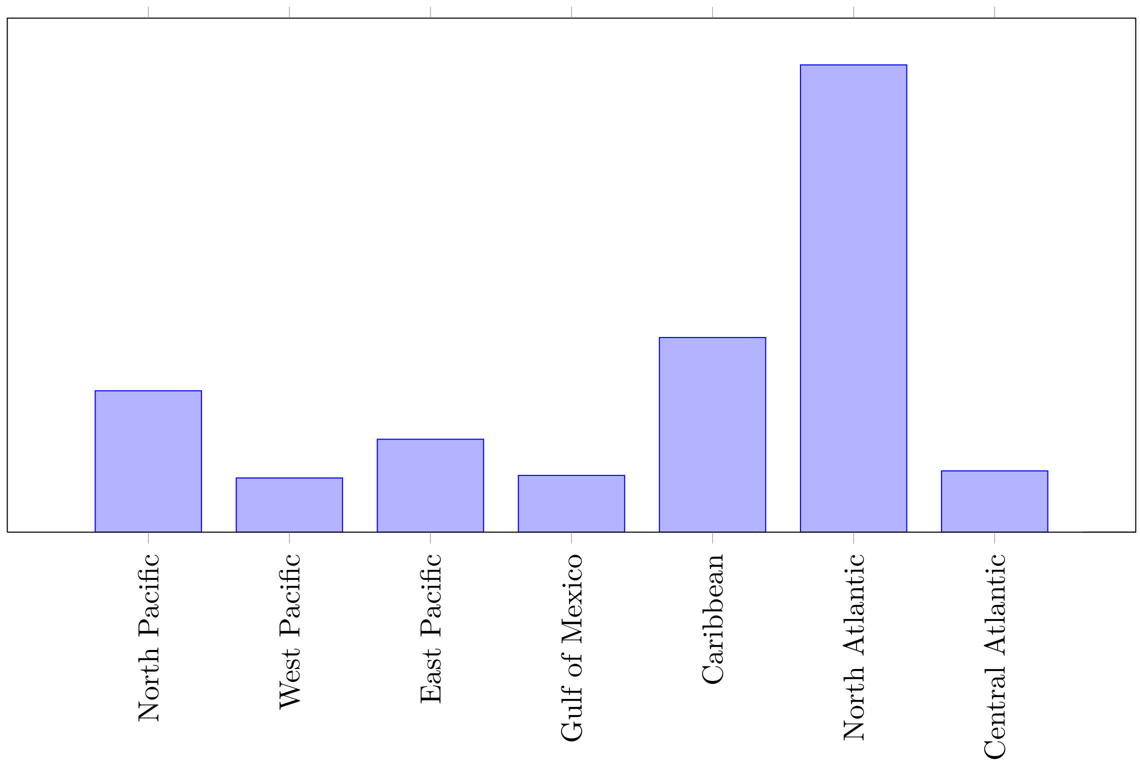


*Figure A-2. Geographic domain distribution.*

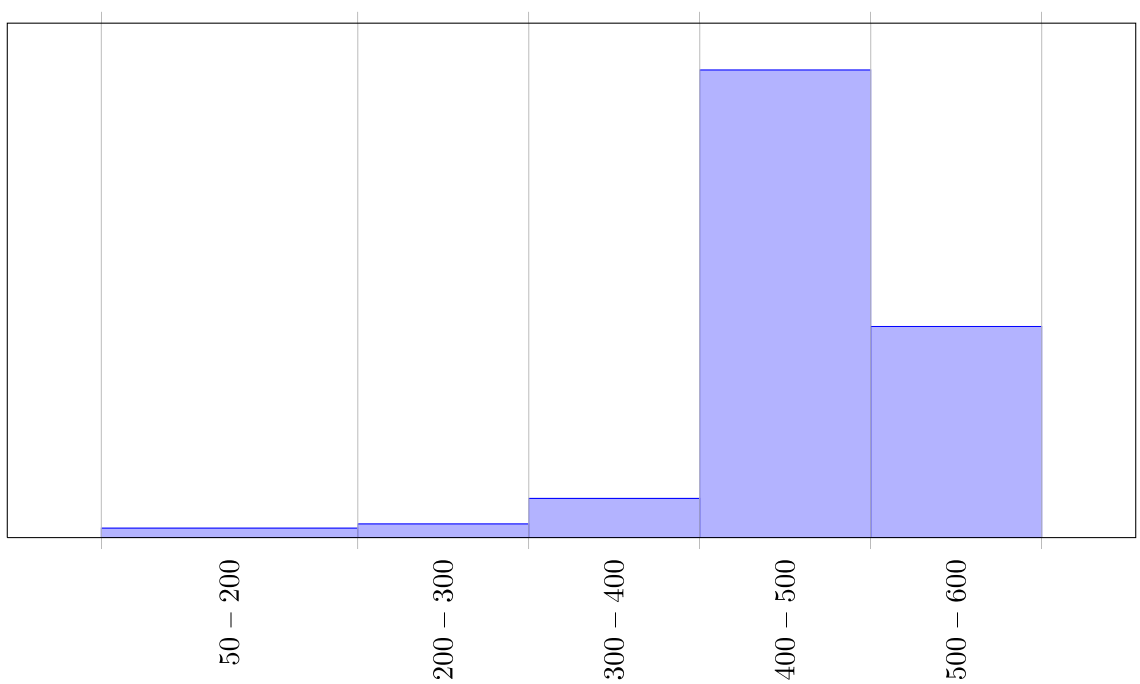


*Figure A-3. Airspeed distribution (kts).*

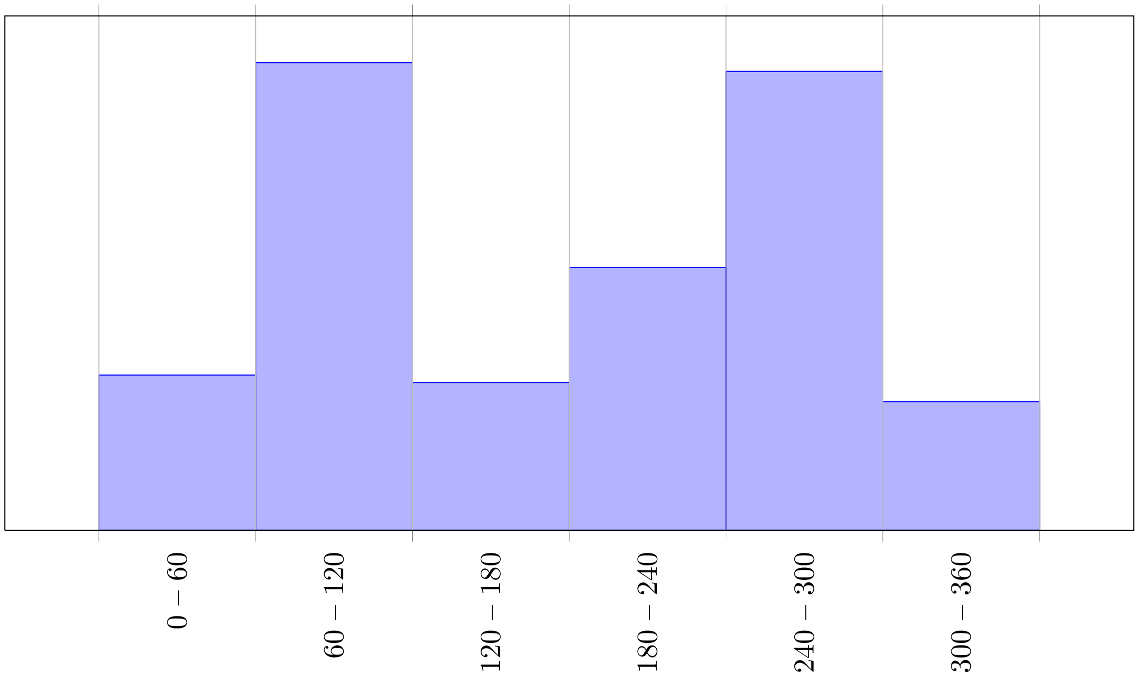


*Figure A-4. Heading distribution (°).*

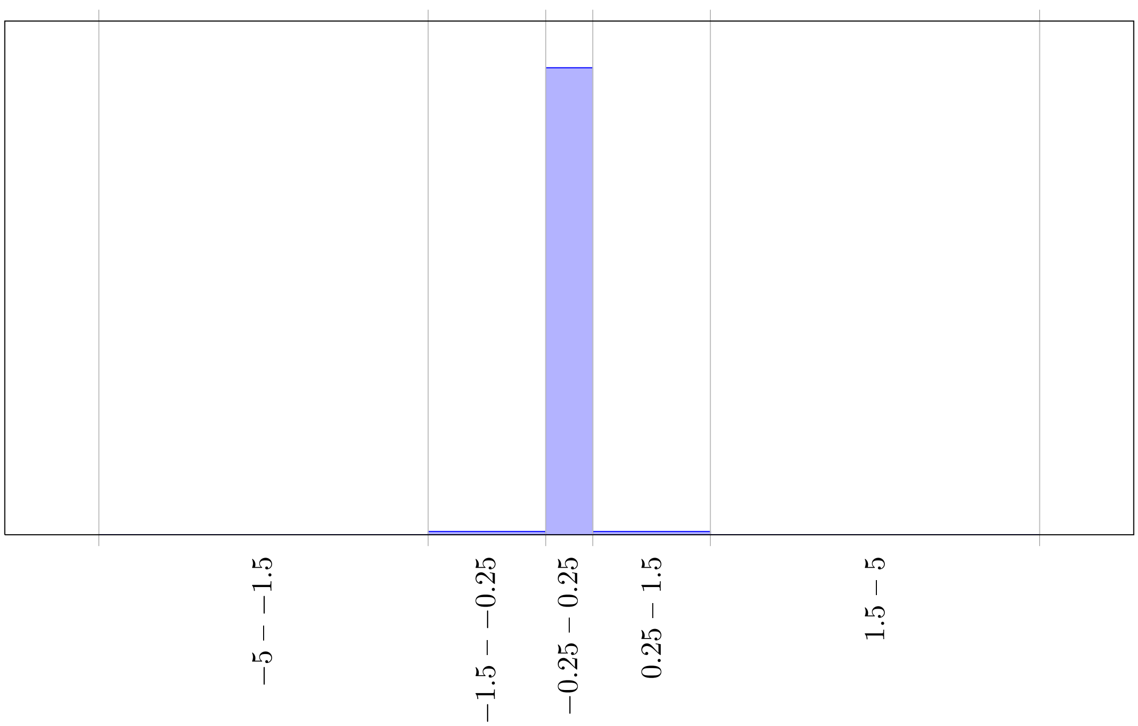


*Figure A-5. Acceleration distribution (kts/s).*

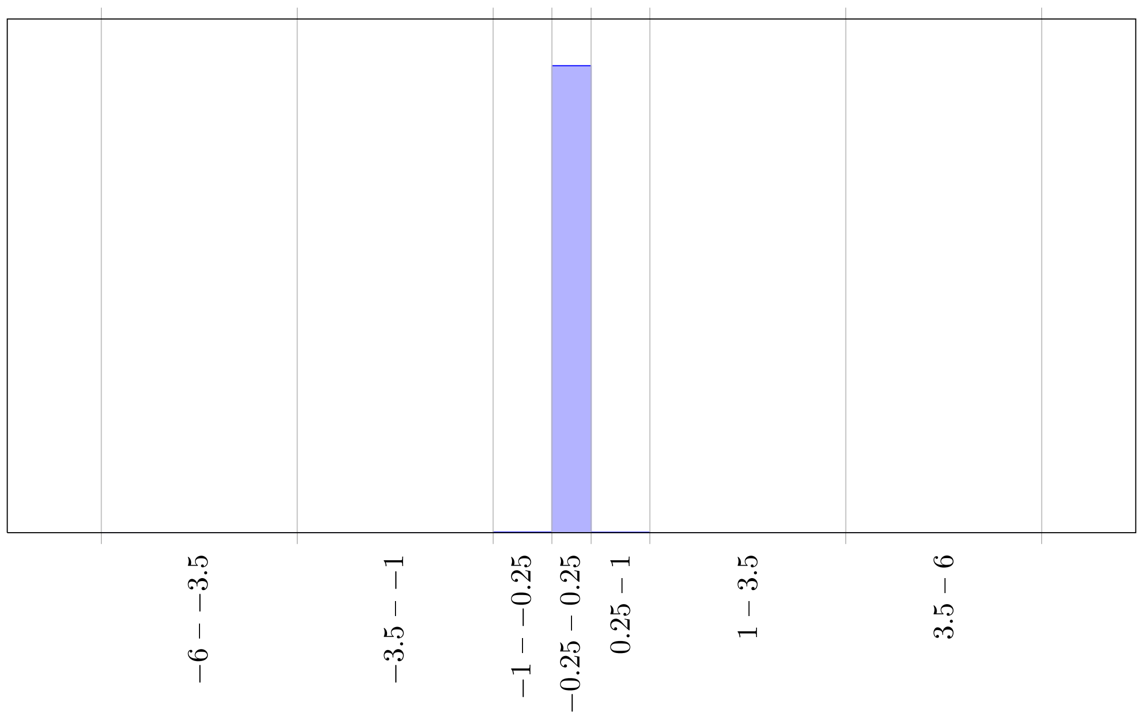


*Figure A-6. Turn rate distribution (°/s).*

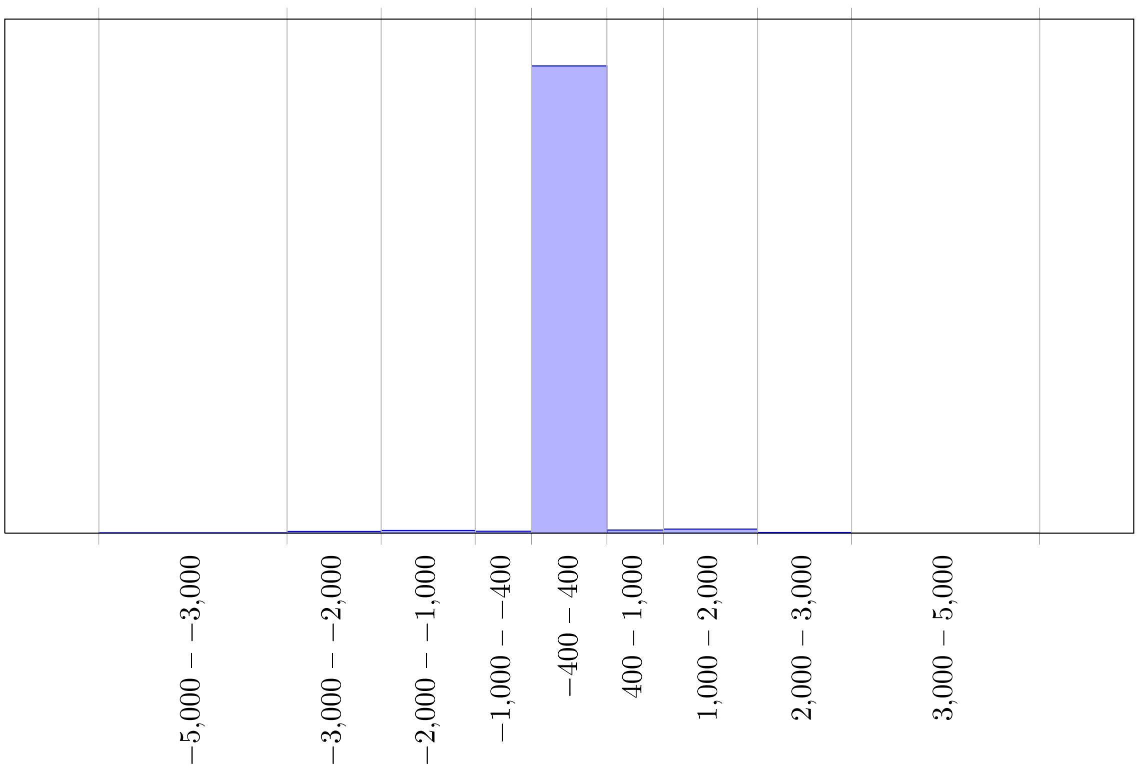


*Figure A-7. Vertical rate distribution (ft/min).*

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# APPENDIX B
# VALIDATION OF ETMS DATA

Previous encounter models have been built using raw radar data that estimate the position of aircraft once every 5 or 12 s. In order to build previous encounter models, the raw radar data are processed to extract tracks. Outliers are removed from the tracks, then the tracks are smoothed and interpolated. Finally, variables in the encounter model are estimated.

The due regard encounter model, that is the subject of this report, is built using ETMS data. There are some important differences between ETMS data and radar data. First, ETMS data includes processed observations of aircraft that are reported once a minute. Second, the ETMS data includes estimates of aircraft ground speed and heading. The differences between radar data and ETMS data raise questions about the suitability of ETMS data for building an encounter model. This Appendix describes a validation of the ETMS data.

Section B.1 describes the approach to validate that the ETMS data feed can be used to build an encounter model. Section B.2 discusses the results.

## B.1 APPROACH

The validation approach is shown in Figure B-1. First, both radar and ETMS data are collected. Because the purpose here is to validate ETMS data for building a due regard encounter model, tracks were selected near the AN/FPS-117 BAR radar in Barrington, Nova Scotia, Canada. It is a long range radar with a maximum range of approximately 250 NM and has coverage of a portion of the NAT. This radar is located at $43°27'7''$ N and $65°28'20''$ W. Once tracks within 250 NM of BAR are selected from the Radar Evaluation Squadron (RADES) and ETMS data feeds, the tracks are associated with each other. Any track that is found in both data sets is kept and all other tracks are removed. Next, encounter models are constructed from both sets of data and the two models are compared. In total, the two models were each built with 1775 hours of flight data.

Figure B-2 shows a 5-minute window of tracks near the BAR radar. The red marks indicate tracks from the RADES dataset, while blue marks indicate tracks from the ETMS tracks.

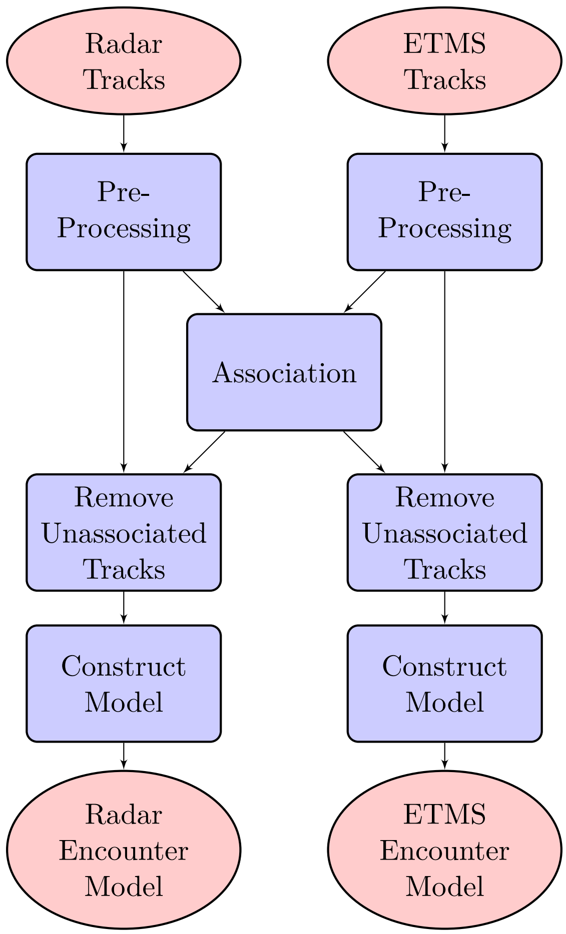


*Figure B-1. ETMS validation process.*

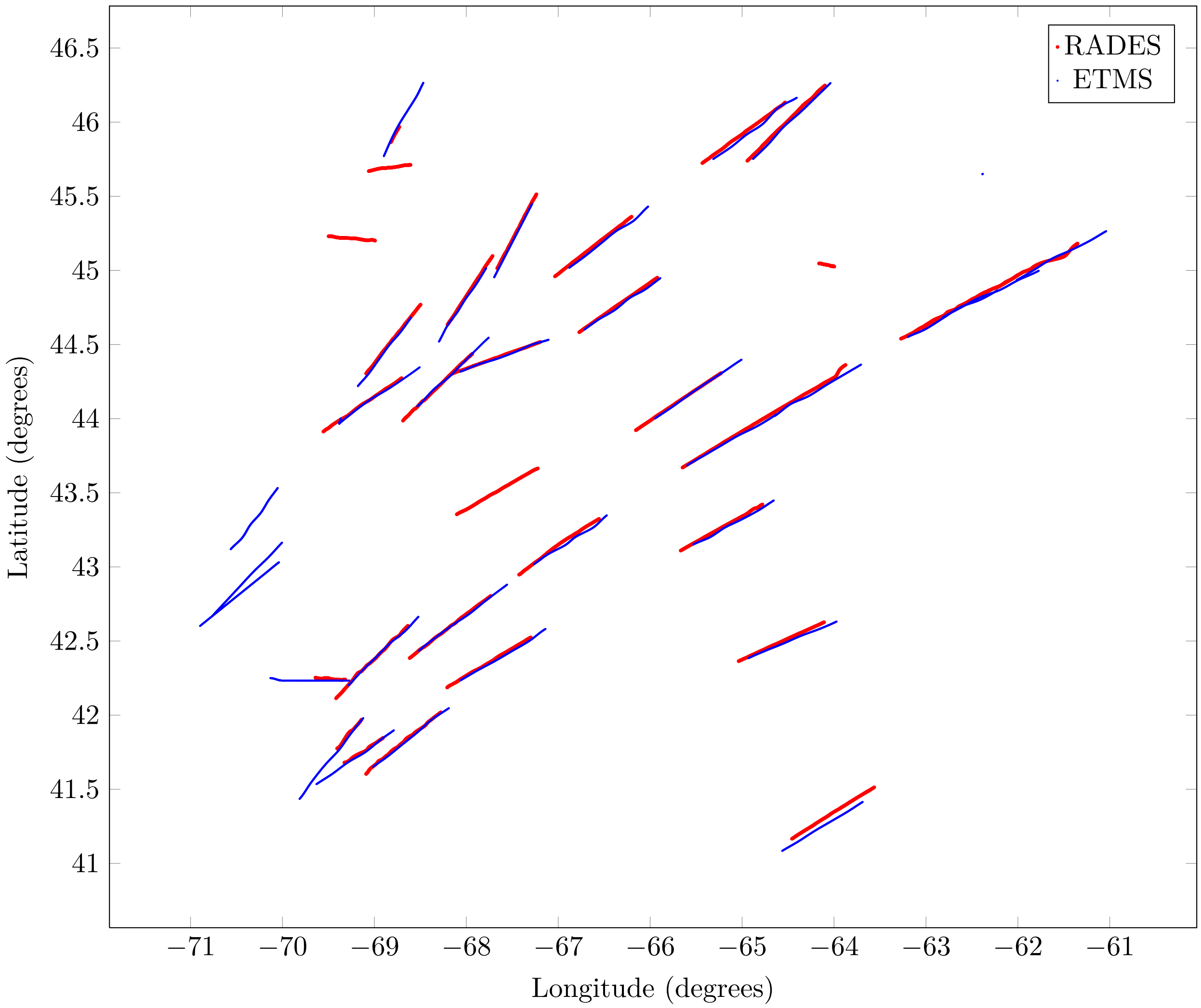


*Figure B-2. Five minute window of tracks.*

## B.2 RESULTS

To compare the two encounter models, the Bayes' similarity scores was calculated, which is a measure of the similarity of two Bayesian networks [7]. If the similarity score is positive, then the two networks are likely the same. However, if the score is negative, then the two networks are likely different. Table B-1 shows the validation results. These results indicate that both networks are different. Because the magnitude for the score of the transition networks is lower, the transition networks are more similar to each other.

**TABLE B-1**

**Validation results.**

| | |
|---|---|
| Initial Network | −186441 |
| Transition Network | −564 |

Figures B-3 through B-6 show marginalized distributions for airspeed, airspeed acceleration, turn rate, and vertical rate. Distributions for the RADES encounter model are in red, while the ETMS encounter model variables are blue. Further inspection of these variables for both networks show that distribution for these variables are very similar. Even though the networks are statistically different, they are assumed to be sufficiently similar for evaluating a SAA system. Thus, one can conclude that the ETMS data feed is acceptable for building an encounter model.

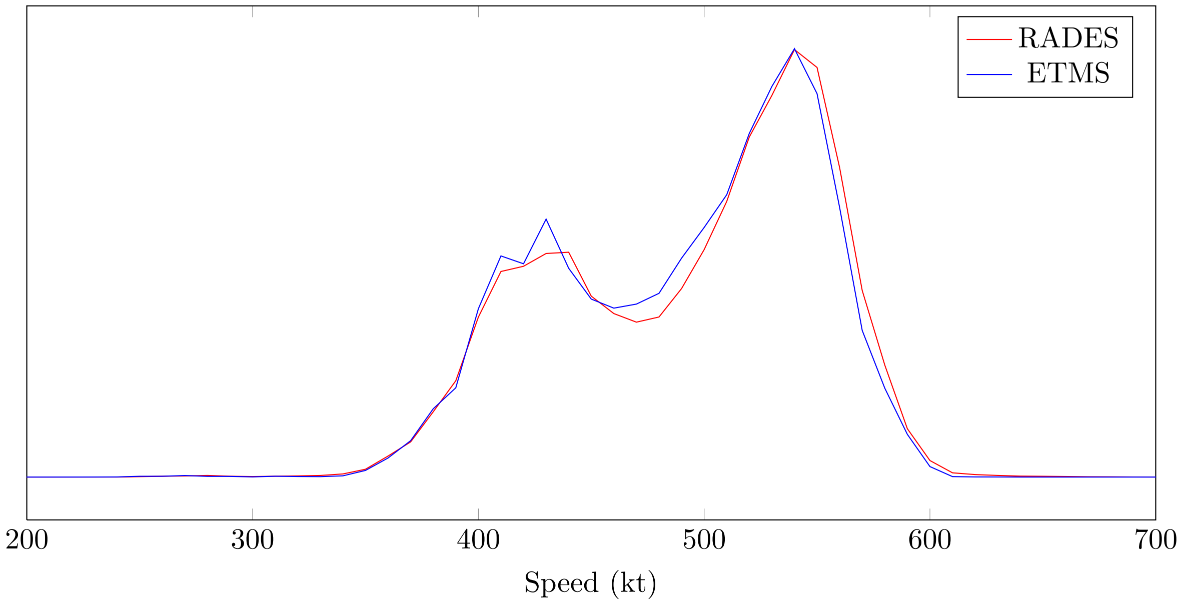


*Figure B-3. Airspeed distribution comparison.*

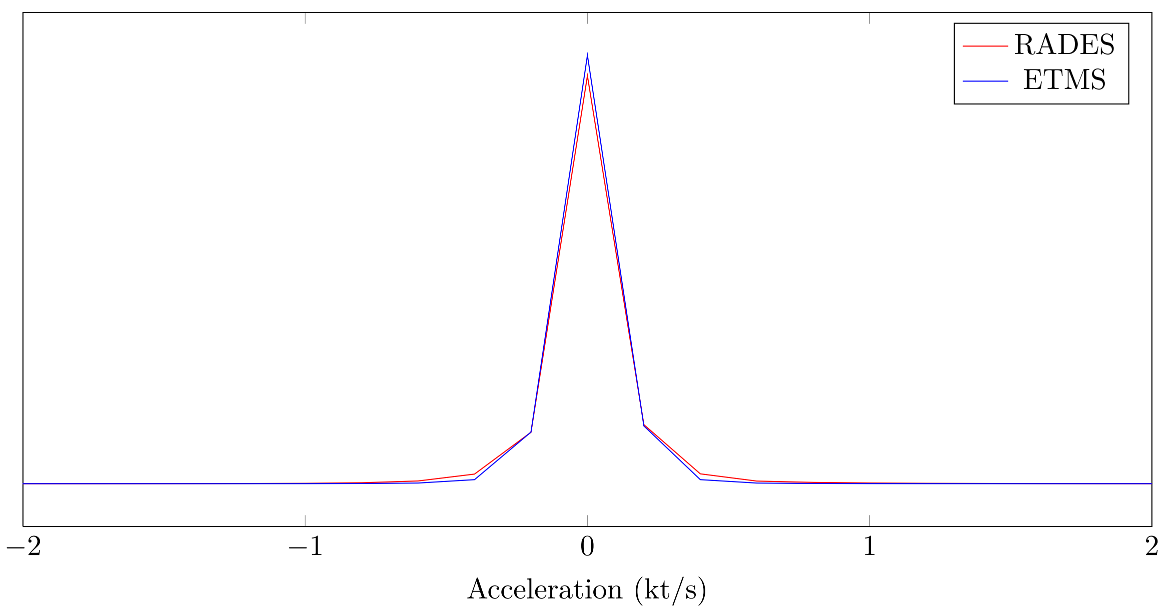


*Figure B-4. Acceleration distribution comparison.*

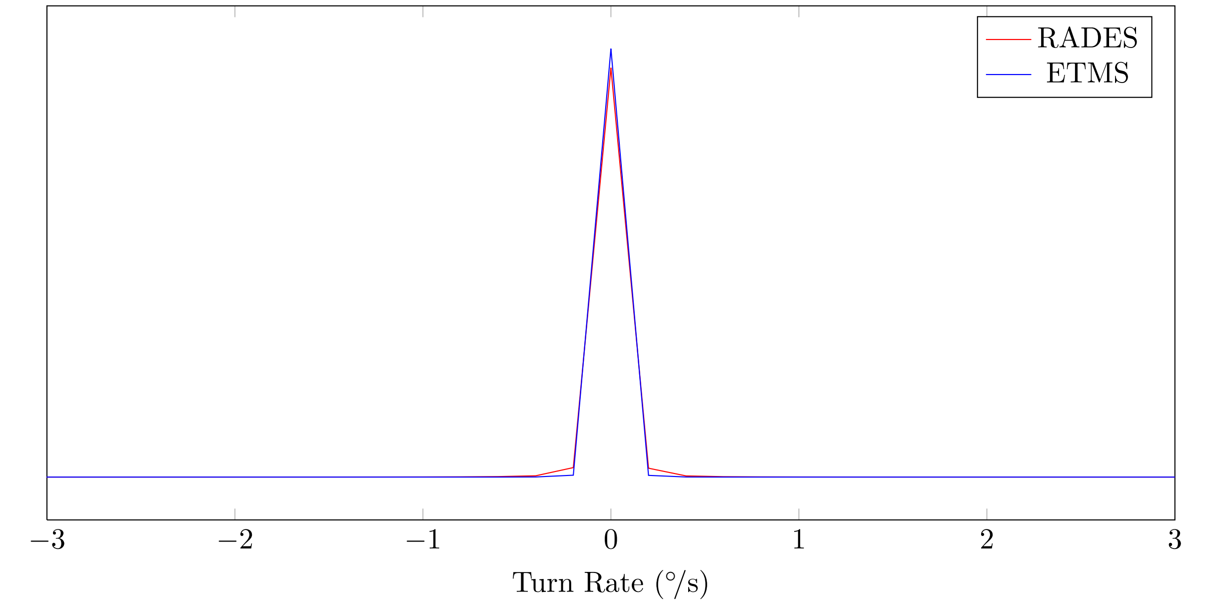


*Figure B-5. Turn rate distribution comparison.*

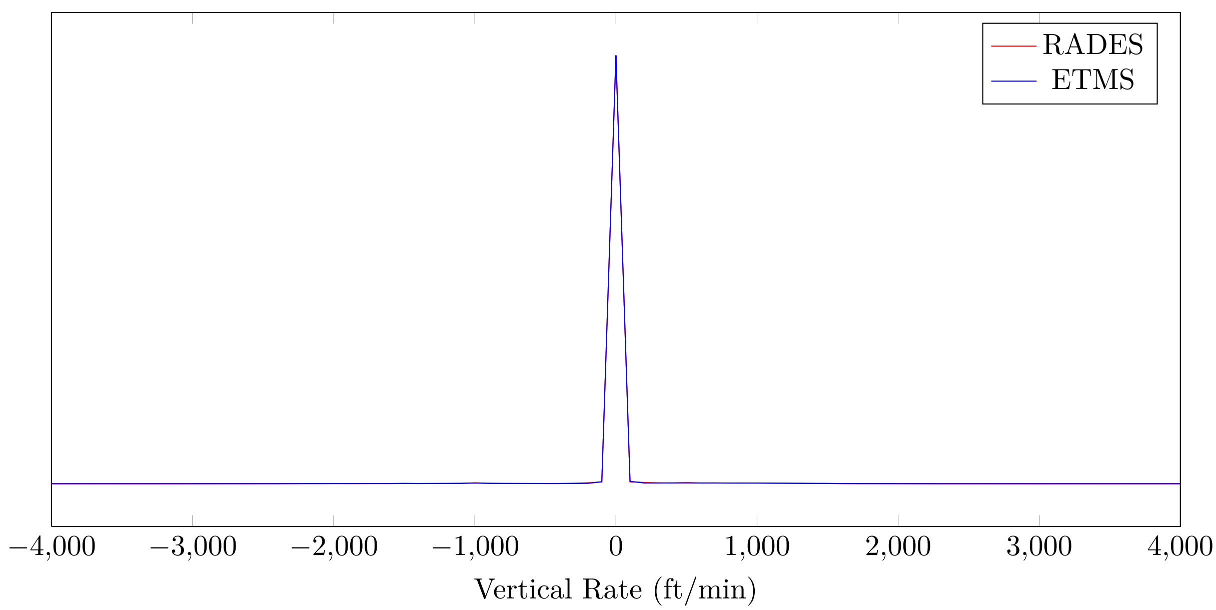


*Figure B-6. Vertical rate distribution comparison.*

# APPENDIX C
# MODEL VALIDITY LENGTH

The model validity length (MVL) is defined as the length of time that simulated tracks generated from the encounter model are representative of the observed tracks; it is one method of deducing the stochastic stability or "validity" of the model. Validity is assessed by comparing the marginal feature distributions (e.g., airspeed, acceleration, and turn rate) of sampled tracks to the observed distributions. Airspeed is important to analyze because it is not explicitly defined in the model's transition network, but is rather propagated in a dynamic simulation by the airspeed acceleration. Because the airspeed is the time integral of the acceleration, it is unbounded unless limits are enforced in the dynamic simulation environment.

## C.1 APPROACH

This process to determine the MVL starts by sampling tracks from the encounter model. The aircraft trajectories are then propagated using a dynamic model and feature distributions at each time step are captured. Features are then discretized and counted into bins, with bin cutpoints defined as in the model. The result is a time history of the discrete feature distributions for the set of tracks. These distributions are compared to those originally observed in the NAS. The first time step at which a simulated distribution is no longer representative of the observed distribution is considered the MVL. The distributions are visually and statistically compared.

For a statistical comparison, the general goodness-of-fit test is

$$\chi^2 = \sum_{k=1}^{K} \frac{(n_{k,S} - n_{k,E})^2}{n_{k,E}} , \tag{C-1}$$

where $K$ is the number of bins, $n_k$ is the bin count, and the subscripts $S$ and $E$ denote simulated and expected (observed), respectively. The $\chi^2$ distribution has $K$-1 degrees of freedom. As the number of samples increases beyond approximately 10,000, $\chi^2$ also increases so that minor deviations between two distributions may become statistically significant [19]. To overcome this, Hamada et al. suggest using a sample size ($n$) proportional to $K$ (such as $K \approx n^{0.4}$) [20]. In contrast to using a small sample size, the sample size ($N_m$) is artificially reduced when calculating $\chi^2$ to $K^{2.5}$. This modifies the calculation of $\chi^2$ to

$$\chi^2 = N_m \sum_{k=1}^{K} \frac{(p_{k,S} - p_{k,E})^2}{p_{k,E}} , \tag{C-2}$$

where $N_m$ is the modified sample size proportional to $K$, and $p = n/N$ where $N$ is the original sample size.

The p-value resulting from the test is the probability of obtaining the sample's $\chi^2$ test statistic or one more extreme by chance. The closer the p-value is to zero, the smaller the probability that the simulated distribution is similar to the expected distribution observed in the NAS.

Given the time history of p-values, the MVL is defined using a threshold p-value; the time at which the p-value decreases past this threshold is considered the MVL. Though the choice of threshold is somewhat arbitrary, p-values of 0.05 or 0.01 are typical choices; 0.01 is used here. The p-value threshold corresponds to the maximum allowable Type-I error—the probability that the results are due to chance alone. It is important to note that p-values resulting from statistical testing can be misleading and must be interpreted with care [19].

## C.2 RESULTS

Figure C-1 shows the MVL p-values as a function of time. At 1000 s simulation time, the p-value is 0.913. This result indicates that the distribution of airspeeds simulated by the model are very stable. Thus, the encounter model can accommodate simulations of encounters that last at least 1000 s. Figure C-2 shows histograms of airspeed at 200 s intervals generated by the encounter model. These results agree with the findings in Figure C-1.

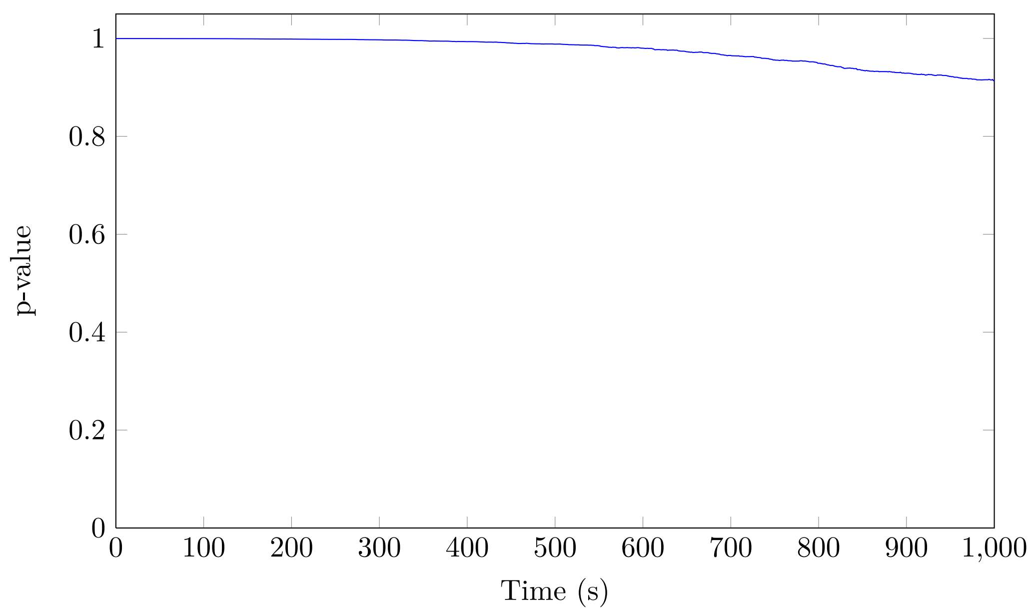


*Figure C-1. Model validity length results.*

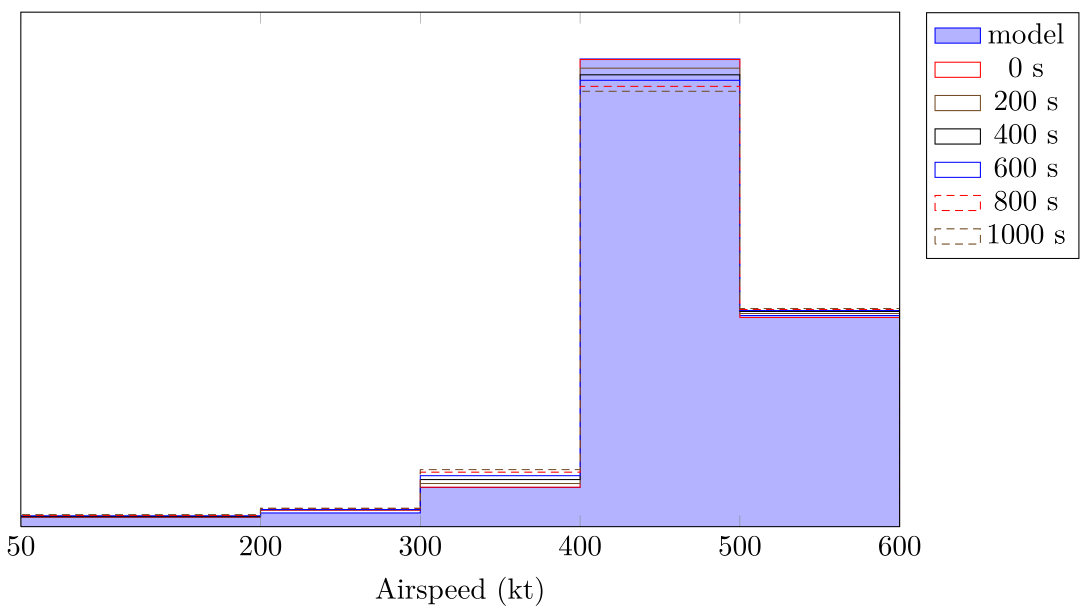


*Figure C-2. Simulated airspeed distributions.*

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**14. ABSTRACT**
Airspace encounter models describe encounter situations that may occur between aircraft in the airspace and are a critical component of safety assessment of sense and avoid (SAA) systems for Unmanned Aircraft Systems (UASs). Some UAS will fly in international airspace under due regard and may encounter other aircraft during these operations. In these types of encounters, the intruder aircraft is likely receiving air traffic control (ATC) services, but the UAS is not. Thus, there is a need for a due regard encounter model that can be used to generate these types of encounters. This report describes the development of a due regard encounter model. In order to build the model, Lincoln Laboratory collected data for aircraft flying in international airspace using the Enhanced Traffic Management System (ETMS) data feed that was provided by the Volpe Center. Lincoln processed these data, and extracted important features to construct the model. The model is based on Bayesian networks that represent the probabilistic relationship between variables that describe how aircraft behave. The model is used to construct random aircraft trajectories that are statistically similar to those observed in the airspace. A large collection of encounters generated from an airspace encounter model can be used to evaluate the performance of a SAA system against encounter situations representative of those expected to actually occur in the airspace.

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