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TCAS III Bearing Error Evaluation

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16. Abstract

TCAS III seeks to enhance TCAS II by providing resolution advisory (RA) capability in the horizontal plane. Additionally, elimination of nuisance RAs through the use of miss distance filtering (MDF) are sought to make TCAS more compatible within the airspace. Both functions (horizontal RAs and MDF) are enabled with accurate estimates of the horizontal miss distance. TCAS III estimates of miss distance rely on range and bearing measurements derived from intruder aircraft replies.

Large errors in the TCAS bearing measurement can be introduced by the airframe structure and other antennas in the vicinity of the TCAS antenna. These large bearing errors can result in large miss distance estimation errors, which will directly affect the performance of the horizontal RA and MDF operation.

In evaluating the performance of the bearing measurements, measurements of the bearing error were used in a simulation of TCAS III surveillance and collision avoidance functions to assess their effect on performance. The performance was evaluated by examining (1) the expected percentage of horizontal RAs issued, (2) the expected reduction in nuisance RAs by the MDF, and (3) the reliability of the monitoring process during a horizontal RA maneuver.

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EXECUTIVE SUMMARY

The Traffic Alert and Collision Avoidance System (TCAS) III seeks to enhance the capabilities of TCAS II by providing additional encounter resolution in the horizontal plane. The addition of horizontal Resolution Advisories (RA) to the array of RA options is enabled through an estimation of the horizontal miss distance, i.e., the predicted point of closest approach in the geometric plane centered at the TCAS aircraft and parallel to the earth's surface. The degree of uncertainty in the horizontal miss distance estimate is directly related to the bearing rate error. Therefore, TCAS III's ability to resolve encounters in the horizontal plane is limited by the accuracy in the estimation of the intruder's bearing rate derived from bearing measurements of the intruder during an encounter. The bearing measurements can contain relatively large errors due to limitations in the antenna subsystem.

The bearing measurement errors are introduced by antenna pattern perturbations, namely those caused by electromagnetic scattering of the airframe and nearby objects, and result in large errors in the miss distance estimation. These systematic errors are correlated in both bearing and elevation and are highly dependent upon the installed configuration (airframe, nearby objects).

In evaluating the performance of the bearing measurement capability, an assessment was made of: 1) the expected percentage of horizontal RAs issued, 2) the performance of the Miss Distance Filter (MDF), and 3) the ability to successfully monitor the separation progress during a horizontal RA maneuver. Each of these performance measurements was evaluated over a wide variety of airframe, collision avoidance (CAS) logic, and sensitivity level configurations.

For this analysis two CAS logic models were used. The first CAS logic model selects the appropriate RA based on a comparison of the expected separation gains of each valid RA. The second model utilizes a working version of the Minimum Operational Performance Standards (MOPS) based TCAS III pseudocode and all 19 RA evaluation tests.

Using the simple separation model, which was more suited for the encounters generated (i.e., co-altitude and no vertical rates) it was shown that for a typical installation (air carrier airframe and Mode S blade antenna located in close proximity), horizontal RAs would be issued only 9-15% of the time against intruders penetrating the TCAS II defined threat boundary. The reduction in the RA rate by the MDF is shown to be small (< 20%), especially for encounters formed with the parameters of sensitivity level 5 (< 10%).

Additionally, during the evaluation up to 2% of the horizontal RAs issued were the wrong sense caused by the perturbations in the bearing transfer functions. It was shown how an intruder safely passing to one side could be construed as having the oppositely signed miss distance value because of the large systematic bearing errors, and subsequently issuing a wrong sense RA.

Lastly, it was shown that once a horizontal RA is issued, it is nearly impossible to monitor the separation progress using miss distance estimates derived from bearing rate estimates. The uncertainty in the estimated miss distance of the intruder throughout the encounter, coupled with the inherent lag in the bearing tracker, makes it difficult to assess the resolution of a horizontal TCAS III maneuver. Two independent measures were used; one was the CAS logic Resolution

Monitoring function, the other a hypothesis test based on the maximum likelihood estimator. For each measure, two scenarios were utilized on the same set of encounters in order to analyze the ability to monitor the separation progress. The first scenario consisted of the TCAS III aircraft performing the issued horizontal RA, therefore realizing the expected separation gain. The second scenario presented the case where the RA was ignored, and thereby realizing no separation gain.

The CAS logic monitoring function was shown to operate as expected in the presence of a bearing-error-free system, based on the number of times the logic correctly recognized the situation. However, when systematic errors were introduced, the number of incorrect resolutions was significant. In fact, the CAS logic monitoring function could not determine the RA progress, based on miss distance estimates, regardless of whether a separation gain was realized or not. Additionally, initial RAs were modified in only 43% of the "RA ignored" scenario (ideal is 100%) and in as many as 40% of the "TCAS turn" scenarios (ideal is 0%).

The hypothesis test provides the time prior to time of closest point of approach (TCPA) at which a positive determination of the RAs effectiveness can be made. This time is equated to the time at which a proper decision could be made about the RAs progress and whether additional measures should be taken. The results of the hypothesis test again illustrated the difficulty in determining the separation progress. It was shown that for a typical TCAS III configuration, at best there may be 6-8 seconds left before TCPA to modify the initial RA. After accounting for pilot delay (assumed between 2-5 seconds) and aircraft accelerations (4 seconds minimum), there is no time left to resolve the encounter.

It is clear from these analyses that the results are highly dependent upon the TCAS III configuration, and in particular the installation configuration (i.e., the local electromagnetic environment defined by the airframe structure and the type and relative location of nearby objects). Defining a single error model that encompassed all possible TCAS III configurations would be unattainable and incomplete.

The analysis results show that miss distance estimates based on bearing estimates from a typical TCAS III installation are too inaccurate to adequately support horizontal RAs and miss distance filtering effectively.

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1. INTRODUCTION

The Traffic Alert and Collision Avoidance System (TCAS) III seeks to enhance the capabilities of TCAS II by providing additional encounter resolution in the horizontal plane. The addition of horizontal resolution advisories (RA) to the array of RA options is enabled through an estimation of the horizontal miss distance, i.e., the predicted point of closest approach in the geometric plane centered at the TCAS aircraft and parallel to the earth's surface. The degree of uncertainty in the horizontal miss distance estimate is directly related to the bearing rate error. Therefore, TCAS III's ability to resolve encounters in the horizontal plane is limited by the accuracy in the estimation of the intruder's bearing rate derived from bearing measurements of the intruder during an encounter. The bearing measurements can contain relatively large errors due to limitations in the antenna subsystem.

MIT Lincoln Laboratory has been investigating the sources and magnitude of expected bearing errors, and their associated effects on the TCAS III capability and effectiveness. The bearing error sources include both white noise and systematic contributions. Over the past two years, the effort has focused on the effect of systematic error sources such as the airframe structure and other antennas located in close proximity to the TCAS antenna.

TCAS III acquires and tracks intruders using range and bearing measurements. These measurements are further processed by the collision avoidance system (CAS) logic which applies a range test (tau calculation), and horizontal miss distance estimation to determine whether the intruder is a threat and whether a horizontal or vertical RA should be issued.

In evaluating the performance of the bearing measurement capability, a simulation of the TCAS III surveillance subsystem and CAS logic was used to assess: 1) the expected percentage of horizontal RAs issued, 2) the performance of the Miss Distance Filter (MDF), and 3) the ability to successfully monitor the separation progress during a horizontal RA maneuver. Each of these performance measurements was evaluated over a wide variety of airframe, CAS logic, and sensitivity level configurations.

2. BEARING ERRORS SOURCES

The projected horizontal miss distance between two approaching aircraft has been shown to be related by the following [1]:

$$m = \frac{r^2 \omega}{v}$$

where r is the intruder range, v is the magnitude of relative velocity between TCAS and the intruder, and ω is the tracked bearing rate. Given the reasonably accurate measurements of r and reasonable estimates of v, the error in the miss distance estimate is directly proportional to the error in the bearing rate estimate at the output of the tracking filter;

$$\sigma_m = \frac{r^2}{v} \sigma_\omega$$

where σ_m is the error in the miss distance estimate and σ_ω is the error in the tracked bearing rate. Bearing rate estimation errors are due principally to errors in the bearing measurement. Bearing errors are produced by both random (uncorrelated) and deterministic (systematic) error sources.

2.1 RANDOM OR WHITE NOISE ERROR SOURCES

Random errors are primarily associated with receiver noise, Analog-to-Digital conversion (A/D) quantization, and the resolution of the bearing lookup tables. The net contribution of these error sources is generally low (tenths of degrees), except at long ranges where the receiver noise can cause a 1-2 degree 1σ bearing error. Figure 1 shows the receiver noise 1σ bearing error as a function of range based on the receiver signal-to-noise ratio (SNR) for a typical TCAS unit [2].

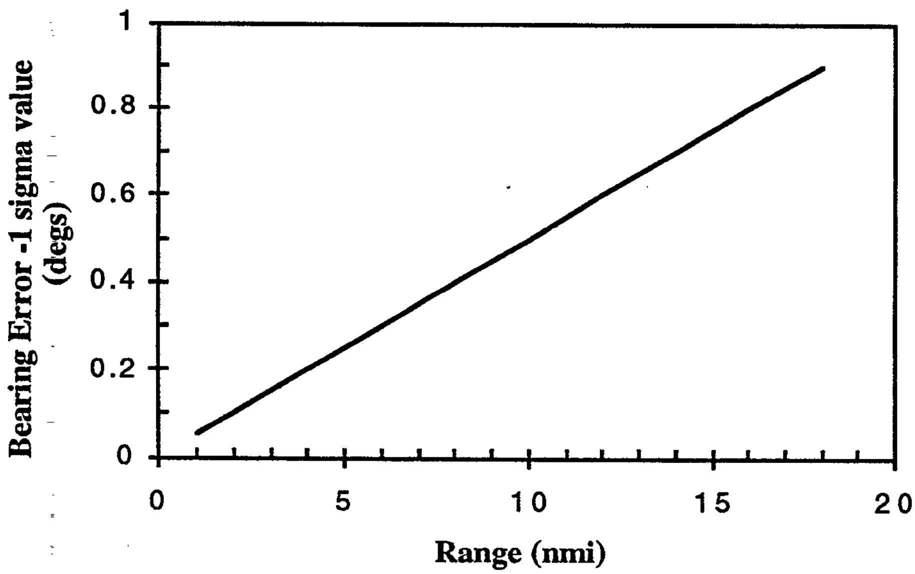

Figure 1. Receiver Noise Error Contribution as a Function of Range.

2.2 SYSTEMATIC ERROR SOURCES

Systematic errors are produced by the local electromagnetic scattering environment as seen by the TCAS angle-of-arrival antenna. Reflections and scattering from the airframe structure (i.e., wings, tail, and engines) and from nearby objects (i.e., transponder and communications antennas) result in systematic bearing errors. Other contributors include antenna pattern variations as a function of elevation angle and differences between the top and bottom antenna pattern structures. All these sources result in bearing biases that are a function of bearing angle, elevation angle and aircraft configuration. Systematic bearing error is characterized by an oscillation in the bearing input/output transfer function. Significant errors can result in the TCAS tracked bearing rate estimate depending on the frequency of the systematic bearing error oscillation. Characterization of actual TCAS III bearing transfer functions for a variety of airframe types and nearby antenna locations was accomplished through the following two efforts.

The Ohio State University (OSU), under contract with the FAA, used a computer-based Geometric Theory of Diffraction program to evaluate the bearing performance of a TCAS III antenna mounted on a variety of airframe types and in the vicinity of various nearby objects [3]. OSU provided numerous bearing transfer functions for three distinct airframe types (the Boeing 727, 737, and 747) and nearby object types (Mode S blade antenna and VHF blade antenna).

Additionally, Lincoln Laboratory conducted a TCAS III antenna test range measurements program to measure the bearing errors associated with various nearby objects in the vicinity of the TCAS III antenna [4]. The objects included a Mode S blade antenna, a VHF communication blade antenna, Global Positioning System (GPS) antenna, and anti-collision light among others. Much of the OSU nearby object data was validated using the Lincoln Antenna Test Range data.

2.2.1 Airframe Structure

The effects of the airframe structure on the bearing measurement capability of the TCAS antenna were computed by OSU using computer-based geometric diffraction theory methods. Electromagnetic scattering from the wings, tail, engines, landing gear, and curvature of the fuselage were evaluated. Bearing error transfer functions for three (3) airframe types, B727, B737 and B747, were examined to provide a reasonable estimation of the optimum TCAS III antenna location.

As expected the resultant bearing error transfer functions are highly dependent upon the configuration of the TCAS III installation. This includes the airframe type, the type and relative location of nearby objects, and the method used to determine the reply signal angle-of-arrival. For example a top mounted antenna on a B727 airframe exhibits relatively large errors in the forward quadrant due to scattering from the engine inlet located in the tail structure, whereas a bottom mounted antenna on a B737 exhibits large error oscillations in all quadrants due to scattering from the wing-mounted engine nacelles.

2.2.2 Nearby Objects

Nearby objects to the TCAS antenna consist of other aviation systems antennas and aircraft strobe lights. In terms of their electromagnetic properties the objects can be divided into two (2) categories; resonant and non-resonant. Resonant nearby objects are antennas operating in the same frequency range as TCAS. The scattering cross section is larger than the optical or physical cross section and therefore the effects remain significant at larger spacings. The mechanism producing reflection is primarily mutual coupling between the TCAS antenna and nearby object radiating elements.

Non-resonant objects include antennas tuned to a different frequency range such as the VHF blade and non-radiating objects such as aircraft lighting. For non-resonant antennas, the mutual coupling between the object and TCAS antenna is small. Non-resonant objects are therefore primarily reflectors scattering the incident energy based on their geometric properties.

Two characteristics in the measured bearing error data were noted during data analysis. The electromagnetic or physical size of the object determines the peak-to-peak error, or "amplitude" of the resulting bearing error transfer function, while the distance between the TCAS antenna and the perturbing object determines the "frequency" of the bearing error transfer function. The resultant sinusoidal curve of the bearing error transfer function along with the tracker parameters define the extent with which the bearing rate estimates are affected. TCAS III is inherently sensitive to some error patterns (around one sinusoidal cycle every 10 degrees), and insensitive to others (constant, slowly varying, and rapidly varying errors). A definition of an "effective bearing error" (EBE) was developed to take these mechanisms into consideration. The effective bearing error value represents a bearing degradation equivalent to that due to an uncorrelated error model of the same RMS value, which can be different than the actual RMS error value of the systematic error.

Figure 2 displays the systematic bearing error of a TCAS III antenna mounted on a B727 airframe with a Mode S blade antenna located 4 feet (1.2 m) aft. Note the large errors in the aft quadrant produced by the tail structure, and the characteristic "chirp" produced by the blade antenna. Included in the figure are the following 1σ error statistics for the forward quadrant: the standard RMS bearing error value, the EBE value, and the 1σ bearing rate error value.

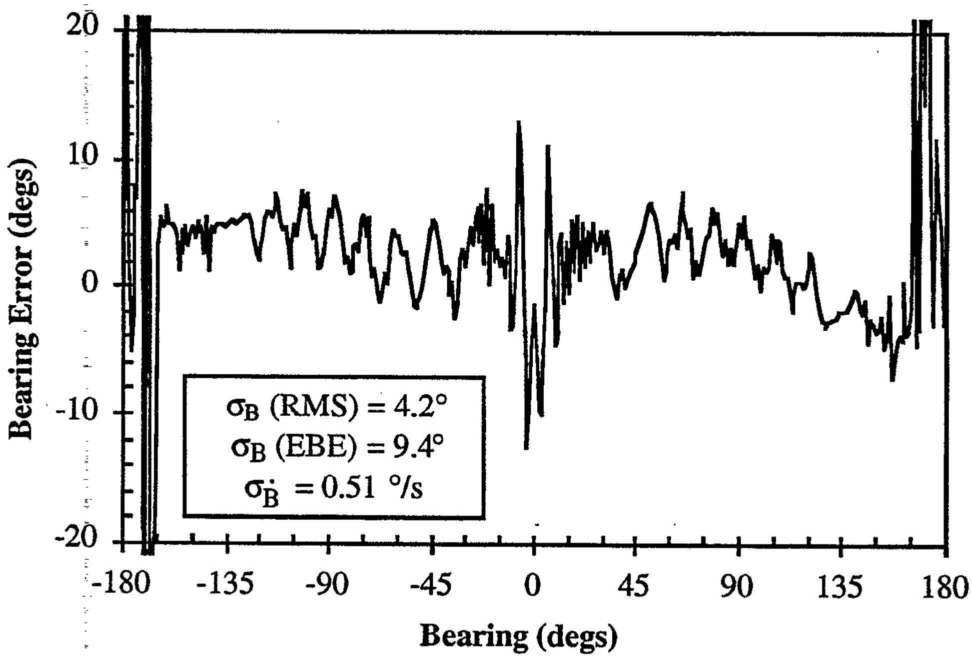

Figure 2. Bearing Error for B727 with Mode S Blade Antenna 4 ft. Aft.

3. SURVEILLANCE SIMULATION MODEL DESCRIPTION

An end-to-end surveillance model was developed to evaluate the effect of both random and systematic bearing errors on the intruder surveillance data provided to the CAS logic for threat detection and evaluation. Specifically, the analysis focused on the effect of bearing error during two phases of the intruder encounter; a) horizontal vs. vertical RA selection at tau time, and b) the RA monitoring function following RA selection. Analysis during the first phase proposes to answer the question, "How often will a horizontal RA be available and selected?". Analysis during the second phase proposes to answer "Now that a horizontal RA has been issued, can TCAS determine its effectiveness?".

Figure 3 illustrates the major functional blocks of the end-to-end surveillance model. The following brief description includes overviews of the three major functions; the Encounter Generator, TCAS III Surveillance, and a representative TCAS III CAS logic. Details of the Encounter Generator and TCAS III Surveillance functions are described in Appendix A. An in-depth description of the CAS logic model will be given here since its characteristics are pertinent to the results.

3.1 ENCOUNTER GENERATOR

An Encounter Generator is used to generate co-altitude, non-vertical rate, encounters with varying miss distances and relative velocities in which the initial conditions are determined in Monte Carlo fashion. One aircraft designated as the TCAS III aircraft, is started at the earth-based coordinate system origin, while the other, designated as the intruder is started well in advance of the range at tau. The encounters are initially structured such that penetration of the threat boundary is assured. Basically the encounters are then progressed according to aircraft linear motion equations and appropriate coordinate transformations. The output of the encounter simulator is true intruder range and bearing relative to the TCAS aircraft.

3.2 SURVEILLANCE FUNCTIONS

The surveillance function introduces the error sources that perturb the intruder range and bearing measurements. As detailed previously, these error sources include receiver noise and systematic contributors. Bearing transfer functions which include white noise as well as the error patterns produced by OSU and MIT L/L range measurements are added to the true position of the intruder. These "new" range and bearing values serve as the input to an XY tracker to derive range rate and bearing rate estimates for threat evaluation and processing. Concurrently, the quality of the bearing rate estimate derived from the XY tracker and provided to the CAS logic is determined by comparison of this data to true bearing rate values provided by an idealized Bearing Rate Accuracy Monitor (BRAM) function, as described in the following section.

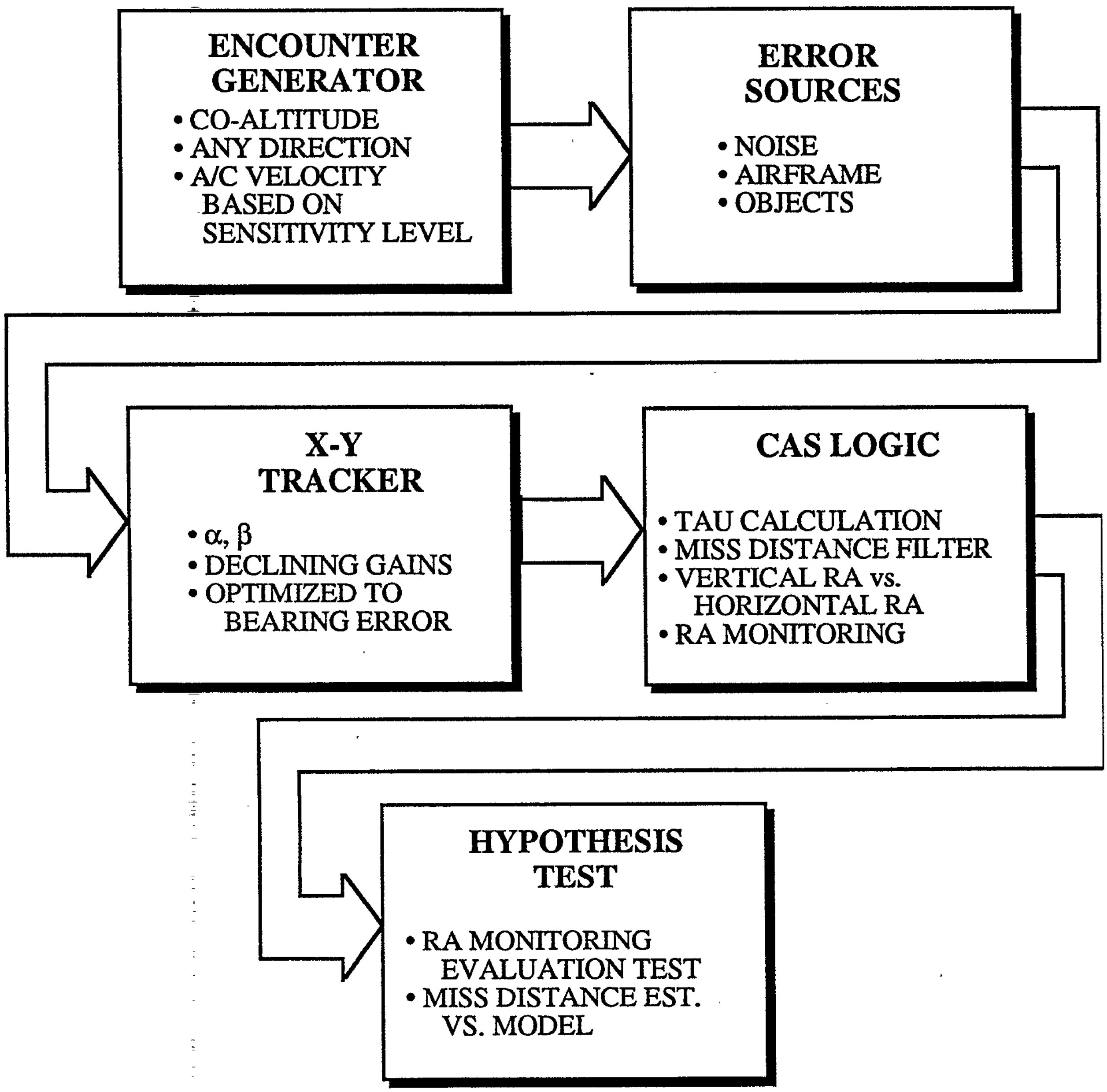

Figure 3. TCAS III Surveillance Subsystem Simulation Model.

3.2.1 Idealized Bearing Rate Accuracy Monitor

The CAS logic requires that the quality of the TCAS III tracker bearing rate estimates be continually monitored in order to assure that the TCAS III horizontal RA selection thresholds are compatible with the installed bearing accuracy. As discussed previously, the accuracies of the bearing measurements and bearing rate estimates are highly dependent upon the installed antenna configuration. In the operational TCAS III, the performance monitoring is carried out by the bearing rate accuracy monitor (BRAM) function [5]. This function takes advantage of targets of opportunity flying straight and level to compare the tracked bearing rate estimates to calculated "true" values based on range and time measurements associated with the a priori knowledge of the target aircraft linear motion. The bearing rate errors for a large number of target tracks are accumulated and the computed error statistics are provided to the CAS logic to be included in the alarm threshold determination. A large number of target tracks are required so that stable error statistics can be provided for all quadrants.

In order to provide for a BRAM estimate for each installed antenna system configuration in the simulation, an "idealized" BRAM function was used. Since the actual systematic bearing error transfer function is known, its error statistics can be readily calculated. The only additional factor that needs to be included is the effect of the specific XY tracker, since its sensitivity to the bearing error transfer function pattern must be determined. The calculation of the BRAM bearing rate sigmas involved an azimuth sweep through the bearing transfer function twice, at a rate of 1 deg/sec, and at a rate of 2 deg/sec. The rate estimates at the tracker output were then compared to the actual rate (1 or 2 deg/sec), the rate errors accumulated and the RMS value calculated for each quadrant. These values were then provided to the CAS logic as BRAM inputs for use in determining the miss distance uncertainties and establishing the appropriate horizontal RA selection thresholds.

3.3 SIMPLIFIED CAS LOGIC FUNCTIONS

In order to assess the effects of any surveillance errors on the performance and effectiveness of a TCAS III system, a suitable representation of the CAS functions must be employed. The CAS functions must include provisions for determining if another aircraft poses a threat and evaluating the effectiveness of the possible escape maneuvers. The functions required to perform this process are numerous and complex as attested to by the number of printed pages of the TCAS III CAS logic pseudocode [6]. Basically, the TCAS III logic extends the TCAS II logic by providing horizontal RA capability in those situations where vertical maneuvers are not as effective. It is expected that vertical RAs will be the predominant choice since most encounters are more naturally resolved with vertical RAs.

The addition of horizontal RAs to the array of RA options is enabled through an estimation of the horizontal miss distance, i.e., the predicted point of closest approach in the geometric plane centered at the TCAS aircraft and parallel to the earth's surface. Inclusion of an accurate miss distance estimation augments the horizontal RA capability by providing a means for reducing the threat boundary to eliminate those intruders which will pass by safely. This threat boundary reduction, called Miss Distance Filtering (MDF), can significantly reduce the alarm rate, if unnecessary threats can be safely excluded from causing alarms.

Based on the position estimates and an assessment of the intruder's anticipated closest-point-of-approach (CPA), the representative CAS logic must provide four main functions; alarm triggering, miss distance filtering, RA selection, and RA monitoring. The CAS logic used in evaluating the bearing error performance is basically a working version of the TCAS III pseudocode functions (Version 3), with some minor simplifications. First, all encounters are performed with a single intruder aircraft (no multiple aircraft encounters), and second, selected RAs do not include; double dimension RAs (i.e., climb and turn left), RA deferrals, preventative RAs, and coordinated RAs.

The logic contains all the necessary functions for performing:

1. Threat detection (Range Test, Tau Calculation and Miss Distance Filtering)
2. Maneuver Modeling
3. Valid RA Determination (Computation of Miss Distances and Error Buffers both horizontal and vertical)
4. RA Selection (RA Evaluation Tests)
5. RA Monitoring and Modification

3.3.1 CAS Logic Models

Intruders that penetrate the threat boundary denoted by the tau circle will be either filtered by the MDF or cause issuance of an RA. If the threat is not filtered by the MDF, then selection of an appropriate escape maneuver must be performed. The selection of an RA consists of:

1. Maneuver Modeling: Modeling of both own aircraft and the intruder for all RA types (Climb, Descend, Turn Left, Turn Right & cross dimension combinations) is performed to provide the associated CPAs.
2. Valid RA Determination: The CPAs for the various maneuvers are modified to account for the estimation errors and adjusted relative to the appropriate positive RA threshold (known as ALIM and RLIM for the vertical and horizontal planes respectively).
3. RA Selection: An RA is selected based on passing a variety of evaluation tests.

Items 1 and 2 are relatively straightforward and are based strictly on the geometry of the encounter. More specifically, by using the tracked Cartesian coordinate position (X,Y) and rate estimates ($\dot{X}$, $\dot{Y}$) the horizontal miss distance is estimated by;

$$\widehat{m} = \frac{X\dot{Y} - Y\dot{X}}{\sqrt{\dot{X}^2 + \dot{Y}^2}}$$

The logic expects to gain additional displacement, Δm, by performing a horizontal maneuver providing a total horizontal miss distance at CPA of $\widehat{m} + \Delta m$.

A similar process is also performed for the vertical plane, that is intruder altitude reports are tracked providing the tracked altitude ($\hat{z}$) and rate ($\dot{z}$). By performing a vertical maneuver, the logic expects additional separation of Δz, giving a total vertical separation at the vertical CPA of $\hat{z} + \Delta z$.

Since selection of the appropriate RA, item 3, requires several evaluation tests to determine the "best" RA to resolve the encounter, the types of selection tests will obviously have a major impact on the RA selection. For this analysis two CAS logic models for RA selection were used. The first is comprised of three tests, namely; a Good Separation Test, an Effective Geometry Test and a Greatest Separation Test. The second model utilizes a working version of the MOPS based TCAS III pseudocode and all 19 RA Evaluation Tests. The following sections describe the two models in more detail.

Simple CAS Logic RA Selection Based on Separation. This CAS logic model selects the appropriate RA based on a comparison of the expected separation gains of each valid RA. This model is an extension to a model used in previous evaluations of TCAS III effectiveness [7][8]. The block diagram in Figure 4 illustrates the selection process and the output data to be accumulated during a simulation run.

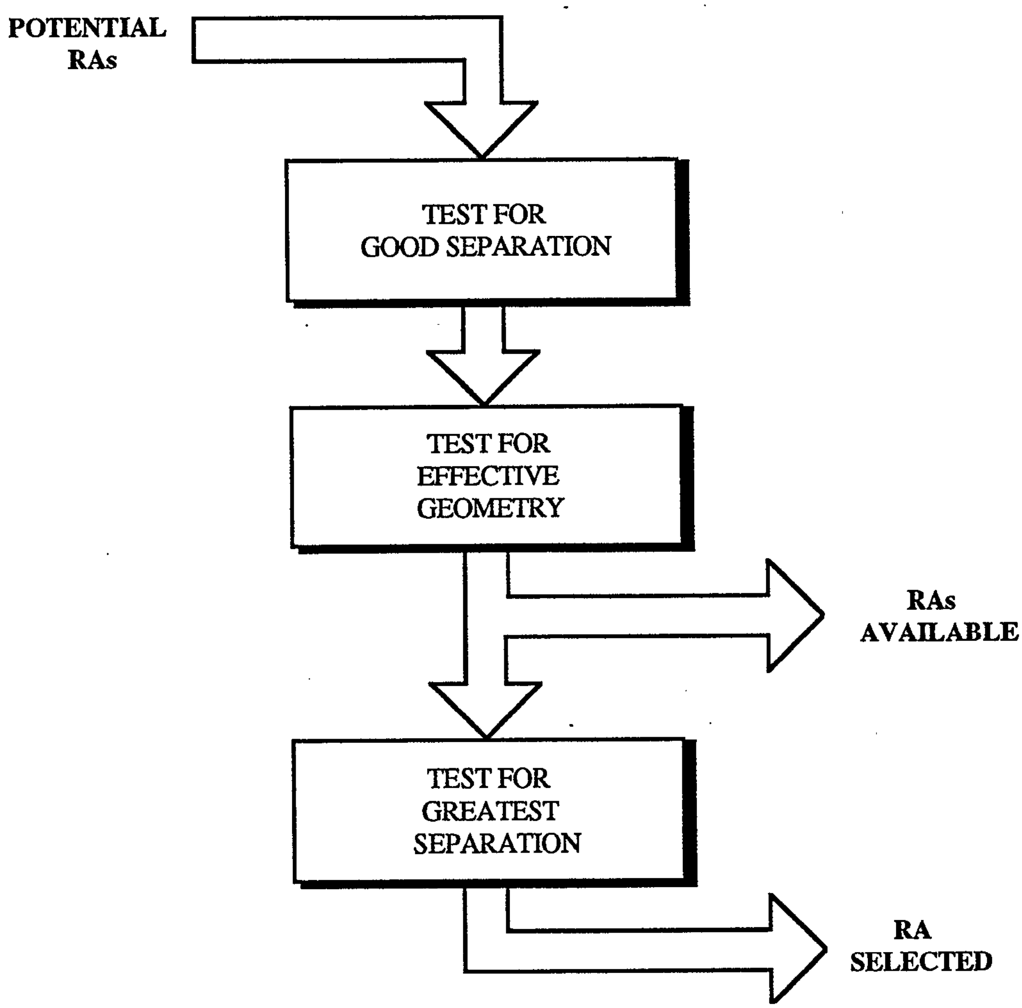

Figure 4. CAS Logic Model Based on Separation.

1. Good Separation Test: This test assures that the RA provides sufficient separation greater than a specified minimum. In other words the RA must satisfy the following inequality, according to RA type:

   Vertical RAs

   $$\hat{z}(t) + \Delta z > c_{min\text{-}z} + \sigma_z$$

   Horizontal RAs

   $$\hat{m}(t) + \Delta m > c_{min\text{-}m} + \sigma_m$$

   where $[\hat{z}(t) + \Delta z]$ and $[\hat{m}(t) + \Delta m]$ are the total expected vertical and horizontal separations respectively, $c_{min\text{-}z}$ and $c_{min\text{-}m}$ are constants based on sensitivity level, and σ_z and σ_m are the 1 sigma error values of the vertical and horizontal miss distance estimates respectively.

2. Effective Geometry Test: This test eliminates horizontal RA types for geometries in which a horizontal RA would be intrinsically ineffective. These geometries are denoted by the angle β, the angle between own aircraft velocity vector and the difference velocity vector. When β is near +/- 180 degrees, a horizontal RA directly affects the miss distance. For β near +/- 90 degrees, the time to CPA is affected but not the miss distance. Horizontal RAs pass this test for the band, $\beta >$ 180+/- 45 degrees. Vertical RAs always pass this test.

3. Greatest Separation Test: This test compares the expected normalized separation gains from the remaining available RAs and chooses the RA providing the greatest separation. The RA chosen during this test is considered the issued RA.

The model illustrated in Figure 4 will determine the percentage of available RAs (i.e., those that meet the minimum separation criteria and pass the geometric test), as well as the final percentage of RA issued.

CAS Logic RA Selection from Version 3 MOPS Pseudocode. This model is essentially a working version of the TCAS III pseudocode provided in the draft TCAS III MOPS. All 19 RA evaluation tests are utilized, including the Good Separation and Greatest Separation tests described above. Since the encounters were constrained to be co-altitude with one intruder aircraft, many of the evaluation tests are not applicable, and some (in particular the "Test for Compatibility with Vertical Rates") even favor horizontal RAs exclusively under these test conditions. Also, the evaluation tests do not include a preferred geometry test, such as the Effective Geometry test above. Nevertheless, results using the MOPS logic is presented for comparison purposes.

4. ANALYSIS AND RESULTS

4.1 HORIZONTAL VS. VERTICAL RA SELECTION

The measurement accuracy of the intruder bearing is taken into account by the horizontal miss distance error buffer (HMD) in the Test for Good Separation. The HMD attempts to compare the predicted horizontal miss distance, derived from bearing rate, against a calculated threshold to determine whether a safe horizontal RA is available for selection (i.e., one that will not turn TCAS III into the path of an intruder).

As shown in 3.3.1.1, the calculated threshold value consists of a constant term and a variable term. The constant term is intended to address the problem posed by slow maneuvering intruders. The variable term buffers against the uncertainty in the miss distance prediction by using independent estimates of the bearing rate accuracy using the bearing rate accuracy monitor (BRAM) function developed for TCAS III. This error buffer is directly proportional to the errors in the bearing accuracy of the TCAS III antenna system. Large errors in the bearing measurement will decrease the effectiveness of horizontal RAs and in fact may preclude their use.

The following sections describe the results of the alarm time statistics accumulated during operation of the simulation, specifically; RA selection results, MDF performance, and one example encounter with a wrong sense horizontal RA selection.

Table 1 provides the parameter inputs into the simulation for the two sensitivity levels tested. All parameters for threat detection and evaluation that are sensitivity level dependent are chosen appropriately according to the inputs from the encounter generator.

Table 1. Simulation Inputs

Sensitivity Level	Aircraft Speeds (kts)		Altitude (ft)
	Maximum	Minimum	
5	250	130	10000
6	600	250	20000

4.1.1 RA Selection Results

Table 2, derived from 50,000 intruder encounters for each bearing error source (including no error and white noise only cases), provides the percentage of time an initial horizontal RA was either available, issued or filtered out at alarm time. Subsequent modifications to the initial RA or the correctness of the initial RA are not considered here, but will be covered in the later sections.

The results are presented according to Sensitivity Level (SL 5 or SL 6), airframe configuration (B727, B737, or B747) and CAS Logic model (simplified or MOPS representation). For each airframe type, several nearby object configurations were tested, and are listed by object type and distance aft of the TCAS antenna. The column labeled σ_ω denotes the 1-sigma bearing rate error value in the forward quadrant determined by the idealized BRAM function (3.2.1) for each bearing error transfer function. Tables 2a through 2f present results exclusively for the simple separation CAS logic model. Table 2g presents results for the CAS Logic Version 3 along with a repeat of Table 2b (2h) for comparison. Note that for typical TCAS III antenna installations involving a nearby Mode S antenna and regardless of airframe type, the percentage of time a horizontal RA can be issued is no more than 15% and more typically around 12%.

Table 2a-h. RA Selection by Simulation Configuration

Table 2a.

Airframe: **B727**
Sensitivity Level: **5**
CAS Logic Model: Simple Separation Model

		Avail	Issued		
Bearing Error Source	σ_ω	%HRA	%HRA	%VRA	%MDF
Error Free	0.0	35	23	46	28
White Noise only	0.04	36	24	50	22
Airframe	0.24	35	19	64	14
Mode S antenna @ 4ft	0.51	22	12	78	7
Mode S @2ft, VHF @ 6ft	0.69	20	12	82	4

Table 2b.

Airframe: **B727**
Sensitivity Level: **6**
CAS Logic Model: Simple Separation Model

		Avail	Issued		
Bearing Error Source	σ_ω	%HRA	%HRA	%VRA	%MDF
Error Free	0.0	26	17	34	49
White Noise only	0.04	30	19	41	40
Airframe	0.24	26	14	57	28
Mode S antenna @ 4ft	0.51	17	10	76	14
Mode S @2ft, VHF @ 6ft	0.69	16	9	81	8

Table 2c.

Airframe: **B737**
Sensitivity Level: **5**
CAS Logic Model: Simple Separation Model

Bearing Error Source	σ_ω	Avail % HRA	Issued %HRA	Issued %VRA	%MDF
Error Free	0.0	35	23	46	28
White Noise only	0.04	36	24	50	22
Airframe	0.05	37	22	52	23
Mode S antenna @ 4ft	0.36	29	15	75	8
Mode S @2ft, VHF @ 6ft	0.67	24	14	81	2

Table 2d.

Airframe: **B737**
Sensitivity Level: **6**
CAS Logic Model: Simple Separation Model

Bearing Error Source	σ_ω	Avail % HRA	Issued %HRA	Issued %VRA	%MDF
Error Free	0.0	26	17	34	49
White Noise only	0.04	30	19	41	40
Airframe	0.05	28	17	43	39
Mode S antenna @ 4ft	0.36	21	11	72	16
Mode S @2ft, VHF @ 6ft	0.67	18	11	83	5

Table 2e.

Airframe: **B747**
Sensitivity Level: **5**
CAS Logic Model: Simple Separation Model

Bearing Error Source	σ_ω	Avail % HRA	Issued %HRA	Issued %VRA	%MDF
Error Free	0.0	35	23	46	28
White Noise only	0.04	36	24	50	22
Airframe	0.08	34	20	60	19
Mode S antenna @ 4ft	0.39	27	14	78	8
Mode S @2ft, VHF @ 6ft	0.67	23	11	85	3

Table 2f.

Airframe: **B747**
Sensitivity Level: **6**
CAS Logic Model: Simple Separation Model

Bearing Error Source	σ_ω	Avail % HRA	Issued %HRA	Issued %VRA	%MDF
Error Free	0.0	26	17	34	49
White Noise only	0.04	30	19	41	40
Airframe	0.08	25	16	49	34
Mode S antenna @ 4ft	0.39	20	11	75	14
Mode S @2ft, VHF @ 6ft	0.67	15	9	84	6

Table 2g.

Airframe: **B727**
Sensitivity Level: **6**
CAS Logic Model: TCAS III MOPS Pseudocode

Bearing Error Source	σ_ω	Avail % HRA	Issued %HRA	Issued %VRA	%MDF
Error Free	0.0	43	43	8	49
White Noise only	0.04	44	44	16	40
Airframe	0.24	42	42	29	28
Mode S antenna @ 4ft	0.51	32	32	52	14
Mode S @2ft, VHF @ 6ft	0.69	24	24	67	8

Table 2h.

Airframe: **B727**
Sensitivity Level: **6**
CAS Logic Model: Simple Separation Model

Bearing Error Source	σ_ω	Avail % HRA	Issued %HRA	Issued %VRA	%MDF
Error Free	0.0	26	17	34	49
White Noise only	0.04	30	19	41	40
Airframe	0.24	26	14	57	28
Mode S antenna @ 4ft	0.51	17	10	76	14
Mode S @2ft, VHF @ 6ft	0.69	16	9	81	8

4.1.2 Miss Distance Filtering Performance

One of the main advantages of having accurate horizontal miss distance estimates is the ability to eliminate RAs to those threats which are known to pass by safely. TCAS II currently issues alarms on all intruders that penetrate a threat boundary defined by the following Range Test inequality:

$$t_{CPA} = -\frac{\left(r - \frac{DMOD^2}{r}\right)}{\dot{r}} < \tau$$

where t_{CPA} is the time to closest-point-of-approach, r is the tracked range, $\dot{r}$ is the tracked range rate, DMOD represents the minimum allowable threat boundary to account for low speed, accelerating intruders and τ is the alarm time. Both DMOD and τ are constants defined by sensitivity level (for SL5, DMOD = 3340 feet and τ = 25 sec). Since the Range Test is a function of the difference velocity ($\dot{r}$ is the component of v lying along the range vector), the maximum horizontal miss distance satisfying the test (or conversely not triggering an alarm) is a function of the aircraft velocity ($\approx$ vT/2). The Range Test therefore provides the initial filter for eliminating unnecessary alarms for aircraft at large horizontal miss distances. However, this filtering can be enhanced even further if accurate estimates of the intruder miss distance can be made.

Miss distance estimates can be derived from estimates of the intruder bearing rate but as has been shown, the error in the miss distance estimate is directly proportional to the error in the bearing rate estimate which is driven by the errors in the bearing measurement. Prior to RA selection, the MDF in the threat logic compares the predicted horizontal miss distance, derived from bearing rate, against a calculated threshold according to the following inequality to determine if the alarm is unnecessary:

$$\hat{m}(t) > c_{MDF} + \sigma_m$$

As in the HMD error buffer in the RA selection logic (3.3.1.1), the calculated threshold value consists of a constant term (c_{MDF}) and a variable term (σ_m), where the variable term is directly proportional to the errors in the bearing accuracy of the TCAS III antenna system. Table 2 illustrates the degradation in the performance of the miss distance filter based on bearing rate estimates as the bearing rate errors increased.

4.1.3 Wrong Sense Selection of a Horizontal RA Due to Systematic Bearing Errors

The RA selection analysis in 4.1.1 assumed that the RA issued was correct. During the course of the evaluation several cases were noted (122 out of the 9465 horizontal RAs for the B727 airframe in SL5) where a wrong choice of horizontal RA sense was made because of the systematic error pattern. This section will examine in detail one particular case where the correct RA was not selected due to the bearing errors introduced by the systematic error pattern.

The particular encounter has the following parameters and TCAS configuration:

	Own Aircraft (TCAS III equipped	Intruder Aircraft
Speed	250 kts	250 kts
Heading	360 degs	180 degs
Altitude	10,000 ft	10,000 ft
Intruder True Miss Distance	1,300 ft (passing to left)	
Aircraft Type	B727	
Nearby Object	None	

Figure 5 shows the track data as the encounter progresses from an arbitrary start time (approximately 30 seconds prior to the alarm boundary) to RA issuance. The true track progresses at a constant projected miss distance of -1300 feet, passing to the left of the TCAS III aircraft at co-altitude. Systematic errors introduced by the airframe structure perturb the surveillance reports such that the TCAS III tracked data indicate an apparent positive bearing rate and therefore a positive miss distance (passing to the right). The positive bearing rate and large positive miss distance estimation result in a TURN LEFT horizontal RA, hence turing the TCAS III aircraft into the path of the intruder.

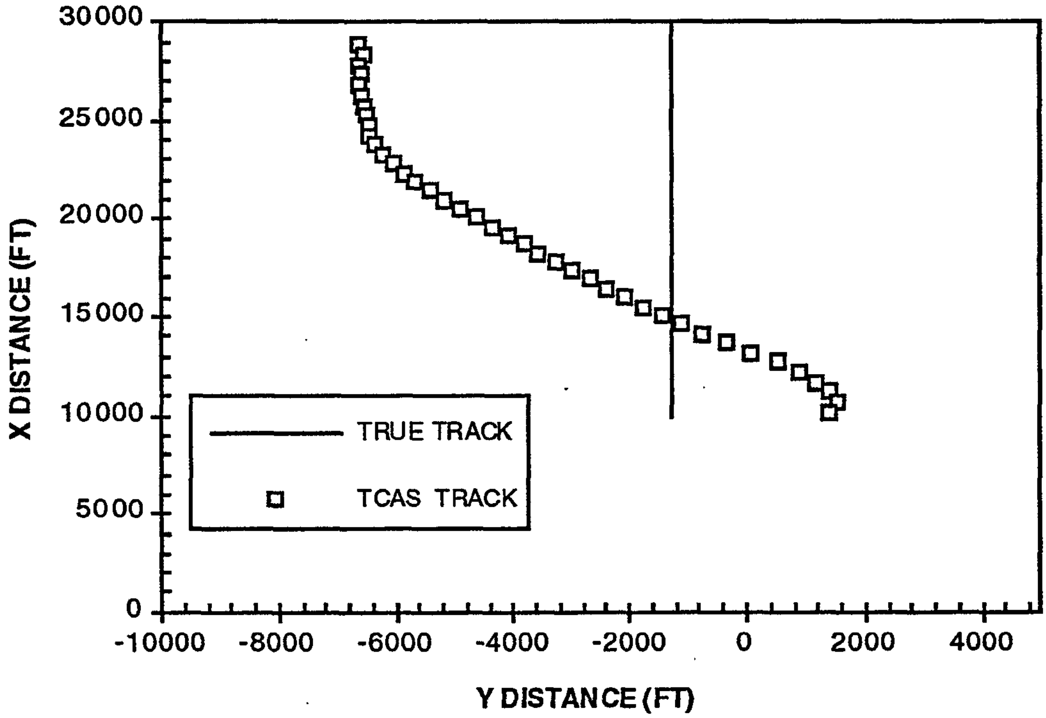

Figure 5. Wrong Sense Selection - B727 Airframe Error Model.

Figure 6 shows the same encounter with errors introduced by a white noise process with the same sigma value as the B727 error pattern. As seen in this figure the alpha-beta tracker is able to properly smooth the track and resolve the encounter correctly (vertical RA issued) because the errors are uncorrelated and not systematic.

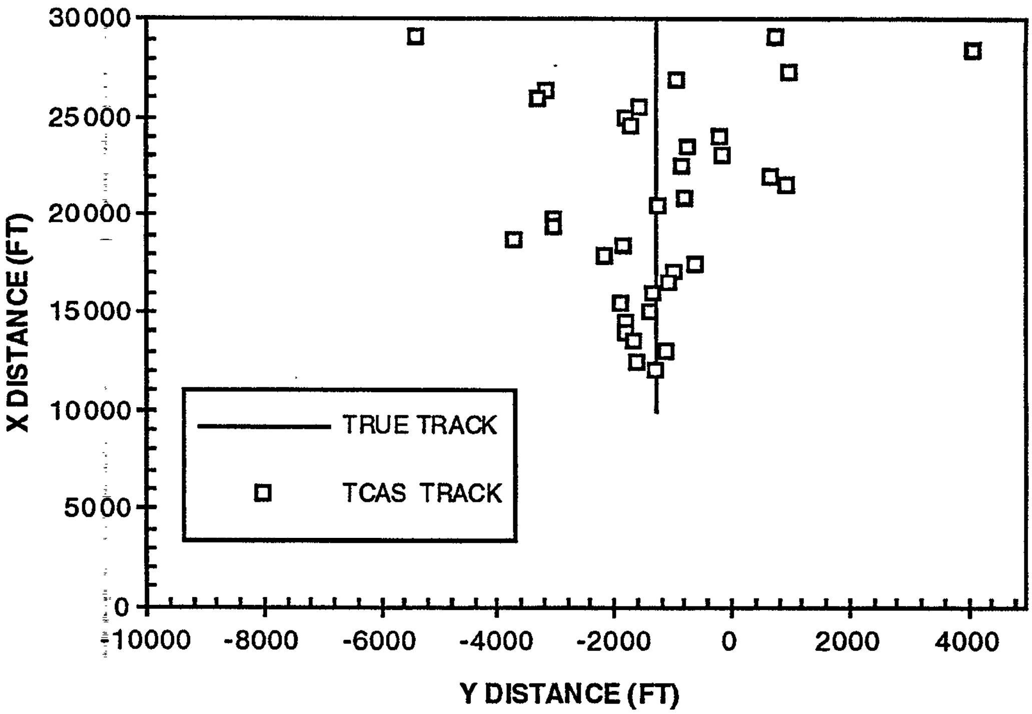

Figure 6. Wrong Sense Selection - Uncorrelated Error Model.

4.2 RA MONITORING CAPABILITY

Under the current design of TCAS III, once a horizontal RA is issued, the separation progress must be continually assessed to determine the success or failure of the RA. The uncertainty in the estimated miss distance of the intruder throughout the encounter, coupled with the inherent lag in the bearing tracker, makes it difficult to assess the resolution of a horizontal TCAS III maneuver.

Two techniques for monitoring the progress of an RA were used to evaluate the bearing performance. The first method is the CAS logic RA monitoring function used in TCAS III. The second is an independent test based on a variation of a maximum likelihood estimation [9]. For evaluating the effectiveness of RA monitoring in the presence of large systematic bearing errors, horizontal RA encounters were analyzed during the time period between RA issuance and time-to-closest-approach (TCPA). During this period the intruder remained along its original course, straight and level, with no accelerations applied. The TCAS III aircraft performed a horizontal 25-deg bank angle maneuver in response to the RA following an initial delay of 6 seconds and 10 deg/sec roll rate. The position measurements and resultant miss distance estimates were then used to assess the recognition of separation gain. Identical sets of encounters were performed for cases where;

1) the TCAS aircraft performed the turn maneuver, therefore a separation gain should have been recognized.

2) the TCAS aircraft ignored the RA and maintained its initial course, therefore no separation gain was realized.

For the first case, the monitoring function should ideally recognize the separation gain, and not attempt to modify or reverse the initial resolution maneuver. In the second case, the monitoring function should recognize early that no additional separation is being attained and therefore provide further action. It should be noted that even in the absence of bearing errors, aircraft accelerations, tracker lag characteristics, system delays, etc., make early recognition of the separation gain difficult.

4.2.1 CAS Logic Monitoring

The TCAS III CAS logic contains a Resolution Monitoring function to monitor the progress of the issued RA and determine its effectiveness. If an increase in separation is not recognized, the logic determines whether further action is required such as modifying the initial RA.

During the RA, the Resolution Monitoring function compares the current normalized miss distance projection at time (t) against a value, S, which is a function of the original normalized miss distance projection computed previously at alarm time (τ) according to the following inequality;

$$\hat{m}(t) - \sigma_m(t) + \Delta m(t) < S[\hat{m}(\tau) - \sigma_m(\tau) + \Delta m(\tau)]$$

where m ( ) is the miss distance estimate, σ_m is the error in the miss distance estimate, and Δm is the expected horizontal displacement caused by the maneuver. If the equation is satisfied, the logic proceeds through a "Quest for Improvement", to determine if a modification to the original RA is required. If another RA is found to provide greater separation, the original RA is modified.

The value of S as a function of the projected miss distance at alarm time (τ) is shown in Figure 7 for SL5.

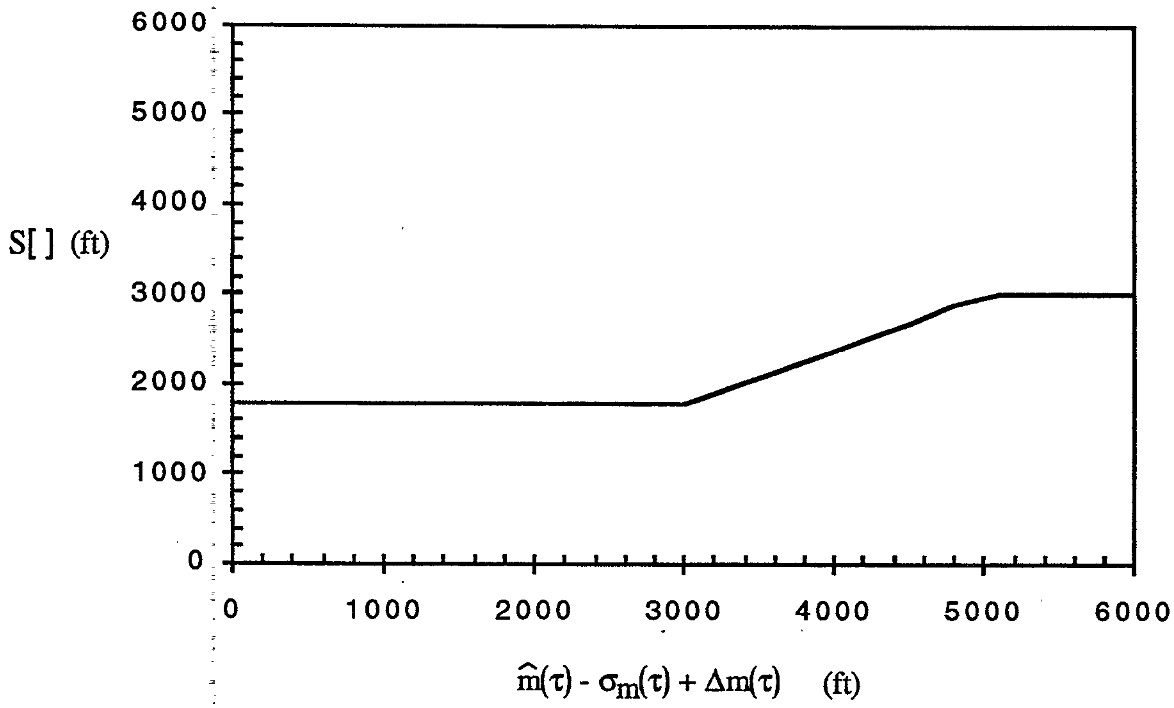

Figure 7. CAS Logic RA Monitoring Threshold - SL5.

In the error free case ($\sigma_m = 0$), when TCAS turns according to the issued RA, the projected miss distance estimate at any time during the maneuver should be larger than the function S. Therefore, in the TCAS turn case no "quest for improvement" is expected. When the RA is ignored, ideally a no gain separation should be immediately recognized. However, due to aircraft accelerations and tracker lag characteristics there is a time delay in the recognition process which would reduce any alternative RA's effectiveness and cause only those encounters with a miss distance of less than 3000 feet to perform this "quest for improvement" process. Figure 8 shows the results of a simulation run with no surveillance errors while monitoring the number and time of the "quest for improvement" for both the TCAS turn and RA ignored cases. The X axis shows the time after the RA issuance and the Y axis is the cumulative percentage of encounters for which a quest was processed. As expected the incidence of quests in the TCAS turn case is negligible. In the RA ignored case, the number of quests is dependent upon the number of horizontal RA-generating encounters with a miss distance of less than 3,000 feet, in this case 40%. For both cases, even though a quest for improvement was processed, none of the initial RAs were subsequently modified.

Figure 9 shows the same results when systematic errors are introduced, namely the B727 airframe bearing error transfer function. The results for both TCAS turning and not turning coincide, with both requiring quest for improvements for over 50% of the RAs. Since both results are very similar, this indicates the difficulty of recognizing a separation gain in the presence of systematic bearing errors. Additionally, of the those encounters processing the Quest for Improvement function, a modification to the initial RA (horizontal to vertical) was made in 80% of the TCAS turning cases and 61% of the RA ignored cases. This equates to an RA modification in 40% and 43% of all horizontal RAs for the TCAS turn and RA ignored cases, respectively.

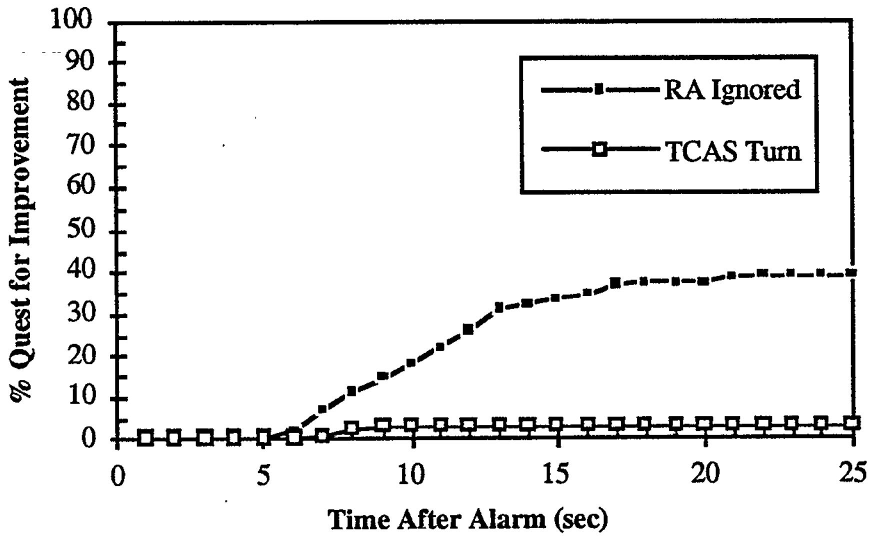

Figure 8. Percentage of "Quest for Improvement" Processed for Error Free Surveillance.

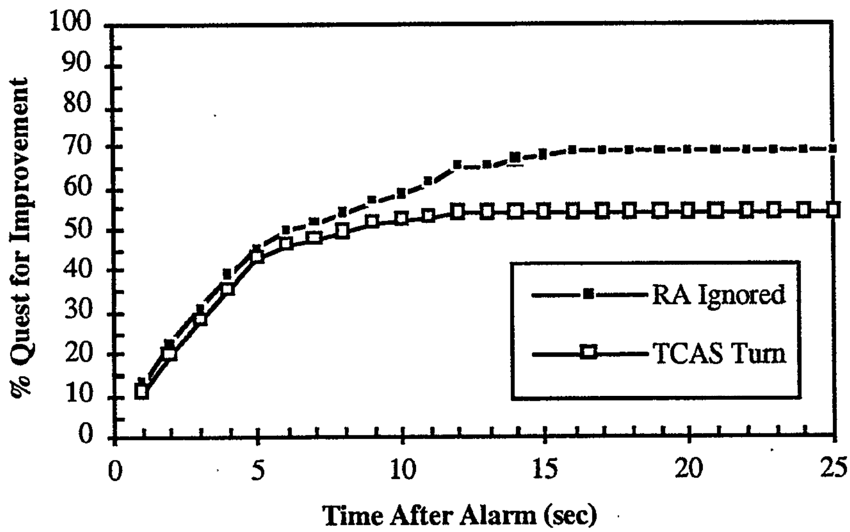

Figure 9. Percentage of "Quest for Improvement" Processed for B727 Airframe Bearing Error Transfer Function.

4.2.2 Maximum Likelihood Estimation Test

As a separate measure of the ability to monitor the RAs progress, a hypothesis test, based on a variation of a maximum likelihood estimation was also used. Basically the hypothesis test defines the earliest time prior to TCPA at which a positive determination of the success or failure of the horizontal RA can be made. The test utilizes simulated encounters to generate intruder position estimates (with measurement errors) and then guesses or hypothesizes the likelihood of one of two possible states based on the current estimate and the a priori knowledge that an RA was issued. The two states are a) own aircraft is performing the maneuver (turning) and therefore providing an increase in separation, or b) own aircraft is not turning and therefore not resolving the encounter. TCAS III relies on the miss distance estimate from the range and bearing measurements in order to make this assessment.

For any point in time after the RA has been issued a declaration of states is made and then compared to the truth. For example, TCAS is turning (truth), but due to the measurements errors, the likelihood ratio hypothesizes TCAS is not turning.

The accuracy associated with an assessment of the effectiveness of a horizontal RA (the hypothesis accuracy) is highly dependent upon the time during the maneuver at which it is calculated. For example, to declare turn or no turn accurately at tau time based on miss distance estimates is nearly impossible, while at TCPA, it is almost certain. There is a time between tau and TCPA where the level of wrong guesses to right guesses is acceptable, which translates into the earliest time (prior to TCPA) that a decision can be made about the RA's progress to resolve the encounter.

Figure 10 shows the miss distance estimates for the error-free case for both the TCAS turn and RA ignored scenarios. The initial miss distance was set to 3000 feet for each of the 100 encounters generated. The result of the hypothesis test is that a positive determination of states cannot be made until 13 sec after the RA was issued or 17 sec prior to TCPA. This delay is primarily due to the pilot delay, aircraft acceleration and tracker lag characteristics.

Figure 11 shows the same encounters using the bearing error transfer functions of the B727 airframe. In this case, the initial miss distance was set to 10000 feet in order to generate a sufficient number of horizontal RAs. Note the overlay of miss distance estimates for both the TCAS turn and RA ignored scenarios over much of the monitoring period illustrating the difficulty in determining the RA progress. The hypothesis test result indicates a positive determination of states at 23 sec after issuance of the RA or 7 sec prior to TCPA.

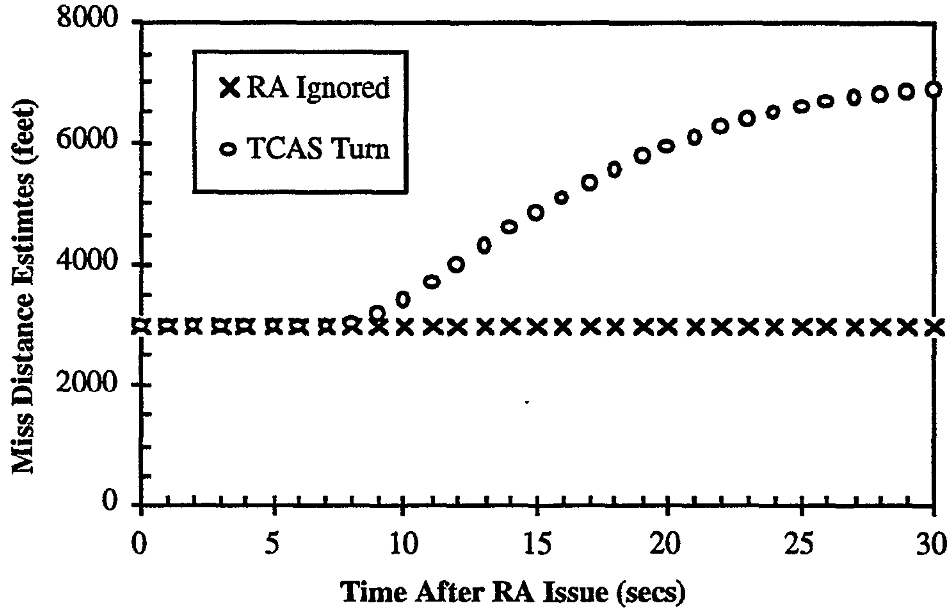

Figure 10. Miss Distance Estimates for Case 1: No Errors.

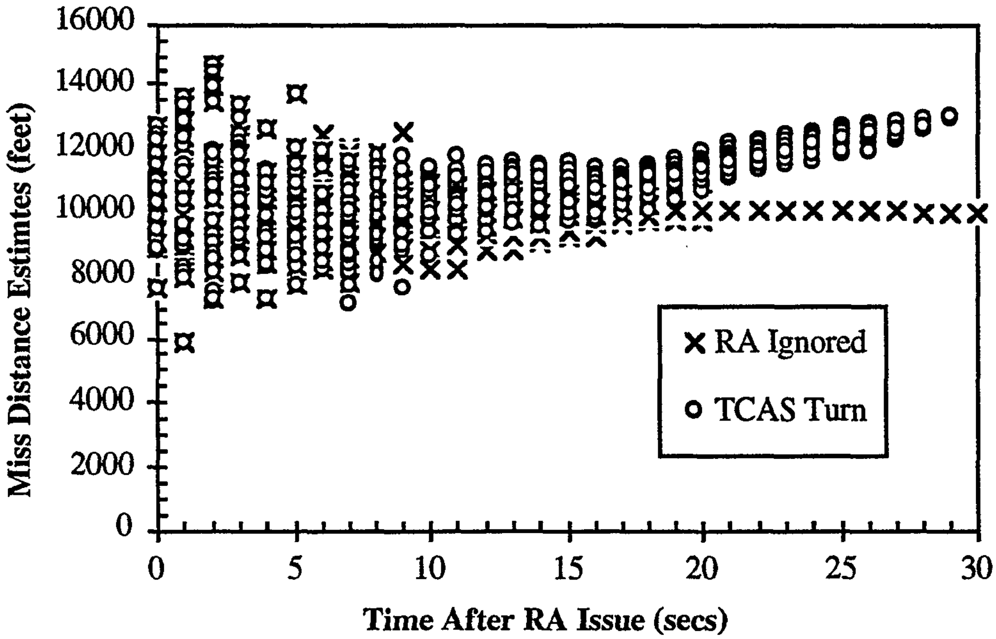

Figure 11. Miss Distance Estimates for Case 2: B727 Airframe.

5. CONCLUSIONS

In evaluating the performance of the bearing measurement capability, an assessment was made of: 1) the expected percentage of horizontal RAs issued, 2) the performance of the MDF, and 3) the ability to successfully monitor the separation progress during a horizontal RA maneuver. Each of these performance measurements was evaluated over a wide variety of airframe, CAS logic, and sensitivity level configurations.

Using a CAS logic model suited for the encounters generated (i.e., co-altitude and no vertical rates) it was shown that for a typical installation (air carrier airframe and Mode S blade antenna located in close proximity), horizontal RAs would be issued only 9-15% of the time against intruders penetrating the TCAS II defined threat boundary. The reduction in the RA rate by the MDF is shown to be small (< 20%), especially for encounters formed with the parameters of sensitivity level 5 (< 10%).

During the evaluation up to 2% of the horizontal RAs issued were the wrong sense because of perturbations in the bearing transfer functions. It was shown how an intruder safely passing to one side could be construed as having the oppositely signed miss distance value because of the large systematic bearing errors, and subsequently issuing a wrong sense RA.

Finally, it was shown that once a horizontal RA is issued, it is nearly impossible to monitor the separation progress using miss distance estimates derived from bearing rate estimates. The CAS logic monitoring function was shown to operate nearly as expected in the presence of a bearing-error-free system, based on the number of times a "quest for improvement" was processed. However, when systematic errors were introduced, the number of processed quests was significant. In fact, the CAS logic monitoring function could not determine the RA progress, based on miss distance estimates, regardless of whether a separation gain was realized or not. Additionally, initial RAs were modified in only 43% of the "RA ignored" scenario (ideal is 100%) and in as many as 40% of the "TCAS turn" scenarios (ideal is 0%).

The results of the independent hypothesis test verified the difficulty in determining the separation progress. It was shown that for a typical TCAS III configuration, there may be at best 6-8 seconds left before TCPA to modify the initial RA. After accounting for pilot delay (assumed between 2-5 seconds) and aircraft accelerations (4 seconds minimum), there is no time to issue another RA.

It is clear from these analyses that the results are highly dependent upon the TCAS III configuration, and in particular the installation configuration, (i.e., the local electromagnetic environment defined by the airframe structure and the type and relative location of nearby objects). Defining a single error model that encompassed all possible TCAS III configurations would be unattainable and incomplete.

Given the results stated above, it is evident that miss distance estimates based on bearing estimates from a typical TCAS III installation are too inaccurate to adequately support effective horizontal RAs and miss distance filtering.

APPENDIX A

TCAS III SURVEILLANCE SYSTEM SIMULATION DESCRIPTION

A.1 INTRODUCTION

This appendix describes the TCAS III surveillance simulation used in evaluating the bearing measurement effects on the effectiveness of horizontal RAs. A block diagram of the major elements of the surveillance portion of the simulation are show in Figure A-1. An Encounter Generator is used to generate co-altitude encounters with varying miss distances and relative velocities in which the initial conditions are determined in Monte Carlo fashion. One aircraft designated as the TCAS III aircraft, is started at the earth-based coordinate system, while the other, designated as the intruder is started well in advance of the range at RA threshold boundary. The encounters are initially structured such that penetration of the threat boundary is assured. Then, the encounter is progressed according to aircraft linear motion equations and appropriate coordinate transformations. By varying the initial condition of an encounter the simulation can be exercised repeatedly producing an unlimited range of scenarios.

The surveillance function introduces the error sources that perturb the intruder range and bearing measurements. These error sources include receiver noise and systematic contributors. Bearing error transfer functions are added to the true position of the intruder. These "new" range and bearing values serve as the input to an XY tracker to derive range rate and bearing rate estimates for threat evaluation and processing. Concurrently, the quality of the bearing rate estimate derived from the XY tracker is determined by comparison of this data to true bearing rate values provided by an idealized Bearing Rate Accuracy Monitor (BRAM) function.

A.2 ENCOUNTER GENERATOR

The encounter generates single aircraft encounters consisting of two approaching co-altitude aircraft. One aircraft is designated as the TCAS III aircraft, or "own," and the other, is designated as the "intruder." Initial headings and velocities for both aircraft are determined randomly using a uniform distribution function. The individual aircraft speed distributions are a function of the sensitivity level at which the encounter is being conducted. Table A-1 provides the parameter inputs into the simulation for the two primary sensitivity levels at which TCAS operates.

Table A-1. Simulation Parameter Inputs by Sensitivity Level

Sensitivity Level	Aircraft Speeds (kts)		Altitude (ft)	Alarm Time (sec)
	Maximum	Minimum		
5	250	130	10000	25
6	600	250	20000	30

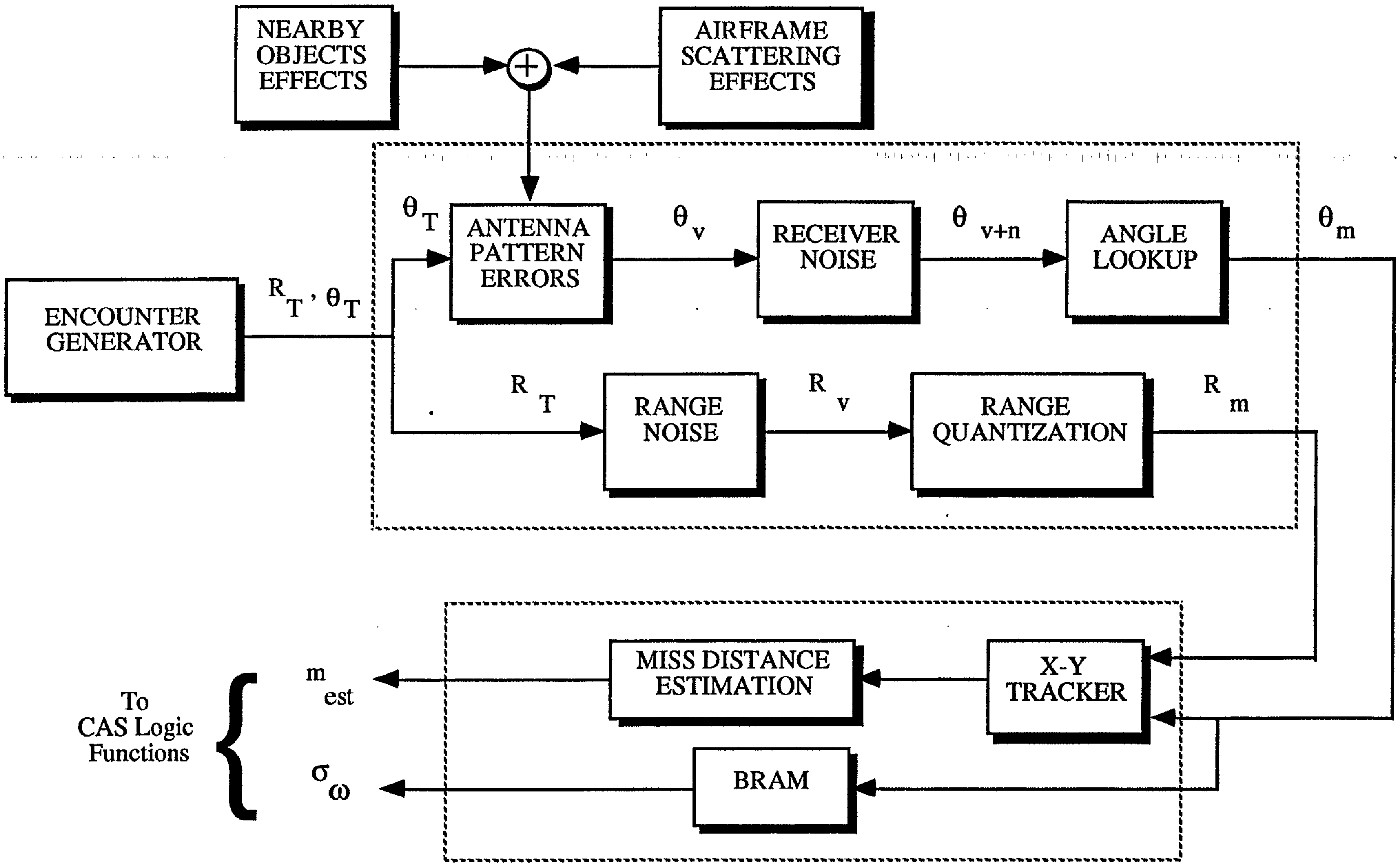

Figure A-1. TCAS III Surveillance Subsystem Simulation - Block Diagram

The relative velocity is calculated based on the geometry of the two individual aircraft velocities, and defines the first order threat boundary;

$$-\frac{v\tau}{2} < m < \frac{v\tau}{2}$$

where v is the magnitude of the relative speed, τ is the alarm time threshold, and m is the miss distance or projected closest point-of-approach (CPA). (Note: this is a first order threat boundary in that the actual boundary is defined by the Bramson criteria which includes an additional factor to protect against small relative velocity encounters).

The encounters are structured such that penetration of this threat boundary is assured by choosing the initial miss distance to be within this boundary. Since the encounter generator produces encounters with varying miss distances and relative velocities, an effective method for describing the aircraft and relative motion is to use the m-vt axis system, where the miss (m) is the horizontal axis and v •time (vt) is the vertical axis. The direction of the relative velocity vector defines the orientation of the coordinate system; the position of "own" aircraft defines the m-vt plane origin. The complete description of the encounter geometry is finalized with the selection of a miss distance. The initial position of the "intruder" aircraft is started well in advance of the range at the RA threshold boundary. All of the appropriate parameters such as the intruder range and bearing can be readily determined. A depiction of an encounter illustrated in the m-vt coordinate system is shown in Figure A-2.

The encounters are then progressed according to aircraft linear motion equations and appropriate coordinate transformations. In the course of an encounter, one or both of the aircraft may be conducting a turn. In this event, the rotational component is taken into account when updating the position and velocity vectors.

The m-vt plane is used initially based on the selected miss and relative velocity to define the encounter geometry; further advancement of the aircraft is performed in an earth-based coordinate system. This is accomplished by keeping an exact account of the intruder relative bearing and the TCAS III aircraft heading. The output of the encounter simulator is the intruder range and bearing relative to TCAS, and the TCAS aircraft state data (velocity, heading, attitude, and altitude).

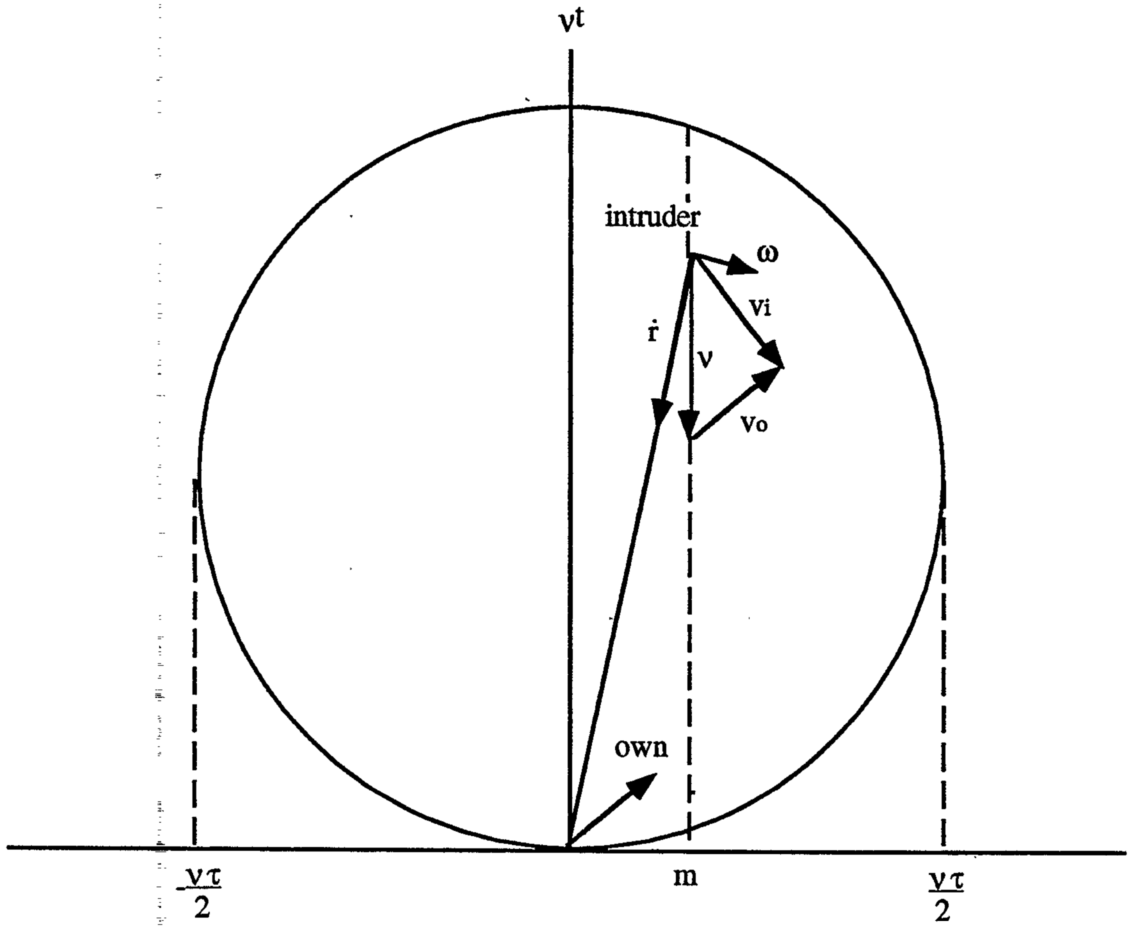

Figure A-2. Illustration of m-vt Encounter Geometry.

A.2.1 Coordinate Systems and Transformations

As noted previously, the aircraft are advance using linear motion in the earth-based coordinate system. This latter system has its x-y plane parallel to the earth's surface, where x points North, y points East. The +z axis points down parallel to the local gravity vector. Each time an encounter is progressed, the aircraft positions in earth-based coordinates are transformed to relative coordinates.

The encounter generator provides range and bearing measurements in the earth-based coordinate system. As inputs to the antenna system the surveillance function introduces the error sources that perturb the intruder range and bearing measurements. The bearing transfer function expects bearing measurements in the aircraft body coordinate system; a system where the x, y, and z axis correspond to the nose, right wing, and the floor direction of the aircraft body respectively. Thus a transformation requiring TCAS III aircraft roll, pitch, yaw, and the earth-based bearing measurement is performed.

These "new" range and bearing values serve as the input to an XY tracker to derive range rate and bearing rate estimates. Because the XY tracker expects bearing measurements in the earth-based coordinate system, a transformation of the perturbed bearing measurement requiring accountability of TCAS III aircraft roll, pitch, and yaw is performed.

A.2.2 Updating Position and Velocity Vectors During a Turn

The TCAS aircraft must have the ability to conduct a turn if the CAS logic issues a horizontal RA. Also, the intruder may accelerate depending upon the statistics of maneuvering aircraft. Consequently, in the course of an encounter, one or both of the aircraft may be conducting a turn. In this event, the rotational component is taken into account when updating the position and velocity vectors.

During its turn, the aircraft velocity rotates at rate w, so:

$$\Delta\bar{v}(t) = \begin{vmatrix} \cos(wt) & -\sin(wt) \\ \sin(wt) & \cos(wt) \end{vmatrix}$$

$$\Delta\bar{r}(t) = \bar{v}(t) \ \text{x} \begin{vmatrix} \dfrac{\sin(wt)}{w} & \dfrac{1-\cos(wt)}{w} \\ \dfrac{1-\cos(wt)}{w} & \dfrac{\sin(wt)}{w} \end{vmatrix}$$

where, $\Delta\bar{r}(t)$ represents the change in position, and $\Delta\bar{v}(t)$ represents the change in velocity.

If a horizontal RA is issued, the model imposes a 6 second delay to invoke the maneuver (one second for TCAS to process the RA and five seconds to reflect the pilot reaction time). Once an acceleration has begun, each aircraft turns at a bank angle of 25 degrees achieved by rolling the aircraft 10 deg/sec.

A.2.3 Markov Motion Model for Unstructured Intruders

A predefined parameter limits the percentage of turning intruders based on a Markov motion model [9]. The Markov motion model defines the percentage of time an aircraft is considered to be either turning or flying straight. This model was determined by observing the motion of a large number of aircraft via ground radar tracks. Basically the model defines the times associated with straight and level flight, and accelerating flight. These times can be translated into statistics associated with straight and turning flight. In other words, an aircraft which is flying straight will have an associated probability that it will transition to a turn on the next scan. Conversely, a turning aircraft will have an associated probability that it will transition to flying straight on the next scan. These two probabilities characterize the Markov process. These probabilities are related to the time duration an aircraft is flying straight or accelerating as follows:

The probability an aircraft will continue to fly straight, given its currently flying straight [p(S/S)] is;

$$p(S/S) = 1.0 - \frac{1.0}{t_S}$$

where t_S is the mean time duration of straight flight as observed in the radar data.

Also, the probability an aircraft will continue to turn, given it's currently turning [p(T/T)] is;

$$p(T/T) = 1.0 - \frac{1.0}{t_T}$$

where t_T is the mean time duration of turning flight as observed in the radar data. These two probabilities can then be used to provide the desired values, that is, the probability an aircraft will begin a turn, given it's flying straight [p(T/S)], and the probability the aircraft will return to straight flight given it's in a turn [p(S/T)]. The desired probabilities are determined by:

$$p(T/S) = 1.0 - p(S/S)$$

$$p(S/T) = 1.0 - p(T/T)$$

The two time durations, t_S and t_T were determined from the radar data to be 100 and 15 seconds respectively. Initially, if the intruder has turning capability, it will either be turning or flying straight based on the following probability:

$$p_i(T,S) = \frac{t_S}{t_S + t_T}$$

Upon each TCAS surveillance scan, the intruder aircraft will maneuver based on the probabilities [p(T/S)] and [p(S/T)]. The turn rate is selected from a triangular distribution centered on a bank angle of 12.5 degrees with limits at 0 and 25 degrees.

A.3 SURVEILLANCE FUNCTIONS

The surveillance function introduces the error sources that perturb the "true" intruder range (r) and bearing (B) generated by the encounter generator. These measured range ($\bar{r}$) and bearing ($\bar{B}$) values serve as the input to an XY tracker to derive range rate ($\hat{\dot{r}}$) and bearing rate ($\widehat{\omega}$) estimates.

A.3.1 Tracker

The intruder is tracked using a recursive declining gain α–β tracker, utilizing higher tracker gains initially and gradually decreasing them over time to the steady state values [6]. The measured positions ($\bar{r}$, $\bar{B}$) from the antenna source are transformed into the earth stabilized system oriented along "own" longitudinal axis in X-Y coordinates, where $x = \bar{r}\cos(\bar{B})$ and $Y = \bar{r}\sin(\bar{B})$.

The positions x and y are tracked separately using identical α–β trackers. The tracker outputs contain estimated position and associated rates. The horizontal miss distance is estimated using the tracker outputs by the relationship;

$$\hat{m} = \frac{x\dot{y} - \dot{x}y}{\sqrt{\dot{x}^2+\dot{y}^2}}$$

where $\dot{x}$ and $\dot{y}$ are the estimated rates of x and y, respectively.

Upon receiving the first position measurement, tracked coordinates are set equal to measured coordinates and the tracked velocity is set equal to zero. When a second consecutive position measurement is received, tracking begins with $\alpha,\beta = 1$. On subsequent scans new values of α,β are calculated and eventually converge to the steady-state values. The following formulas develop the time relationship to generate the declining tracker gains.

$$\alpha = \alpha + G\,(\alpha_s - \alpha)$$

$$\beta = \beta + 0.46\,(\beta_s - \beta)$$

where α_S and β_S are the steady-state values determined by

$$\alpha_s = 0.38\left(\frac{\sigma_B}{T^2}\right)^{-0.28} - 0.06$$

$$\beta_s = \frac{\alpha_s^2}{2 - \alpha_s}$$

$$G = 0.69\,\alpha_s^{1.77} + 0.16$$

where T is the update interval, and σ_B is the standard deviation of the effective bearing measurement error for the installed equipment provided by an idealized BRAM function.

A.3.2. Top-Bottom Antenna Measurement Offset

Surveillance is primarily performed through the "top" antenna pattern as is done in the actual TCAS system to mitigate the effects of ground multipath. Therefore the elevation angle limits for top antenna surveillance extends from -10 to +90 degrees; bottom antenna surveillance is performed for targets at an elevation angle less than -10 degrees. Since the encounter generator produces only co-altitude encounters (as of this writing), surveillance through the top antenna is guaranteed for straight and level flight. However, once a horizontal RA is issued and a turn initiated, surveillance may be switched to the bottom antenna. The bearing transfer functions differ for the top and bottom antenna measurement, and vary as a function of bearing and elevation angle. Switching antennas can result in a large offset in the bearing measurement and may be construed as an acceleration by the tracker adversely affecting the horizontal miss distance estimate.

To compensate for the large differences in the bearing measurements, upon receipt the first position measurement via the bottom TCAS antenna, the tracked coordinates are set equal to the measured coordinates and the tracked velocity is set equal to the predicted velocity; specifically, $\alpha = 1.0$ and $\beta = 0$. That is the predicted position measurements are discarded but the rate estimates maintained and tracking is reinitialized using the first bottom antenna measurements. On subsequent scans, α and β are returned to their previous values.

A.4 IMPORTANT PARAMETER CHARACTERIZATION OF THE ENCOUNTER GENERATOR

As previously mentioned, the encounter generator produces encounters with varying miss distances and relative velocities in Monte Carlo fashion. To generate random encounters, a uniform distribution function is used to select the initial headings and velocities for both aircraft for each encounter. The resultant relative velocity is then used to define the threat boundary in order to determine the miss distance limits within which a miss distance is selected. The miss distance is selected within these limits using a uniform distribution function.

Because of the initial random selection process for the individual aircraft headings and velocities, the resultant relative velocity, and subsequently the miss distance distributions are distinctly not uniform.

A.4.1 Relative Velocity Distribution

The cumulative distribution of Figure A-3 was developed by accumulating the relative velocity values generated by 100,000 runs of the simulation for encounters at SL5. The distribution is seen to increase linearly from low speed encounters (> 50 kts) up to moderate speed encounters (< 370 kts), and tapers off rapidly beyond 400 kts. This indicates that most encounters were equally likely initiated with relative speeds between 50 and 370 kts, with a smaller proportion at the low and high speed ends.

A.4.2 Miss Distance Distribution

The histogram of the miss distance selection for the same 100,000 encounters is shown in Figure A-4. This miss distance distribution is the result of the selection process, namely forcing all generated encounters to penetrate the threat boundary, i.e., keeping $|m| < v\tau/2$ given the distribution of v from A.4.1.

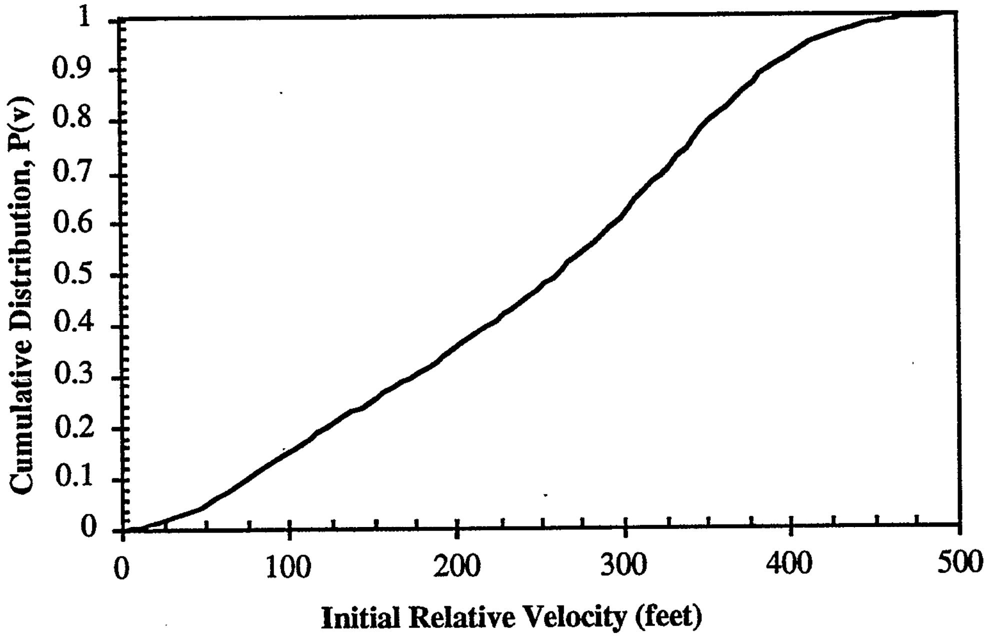

Figure A-3. Cumulative Distribution of Relative Velocity - SL5.

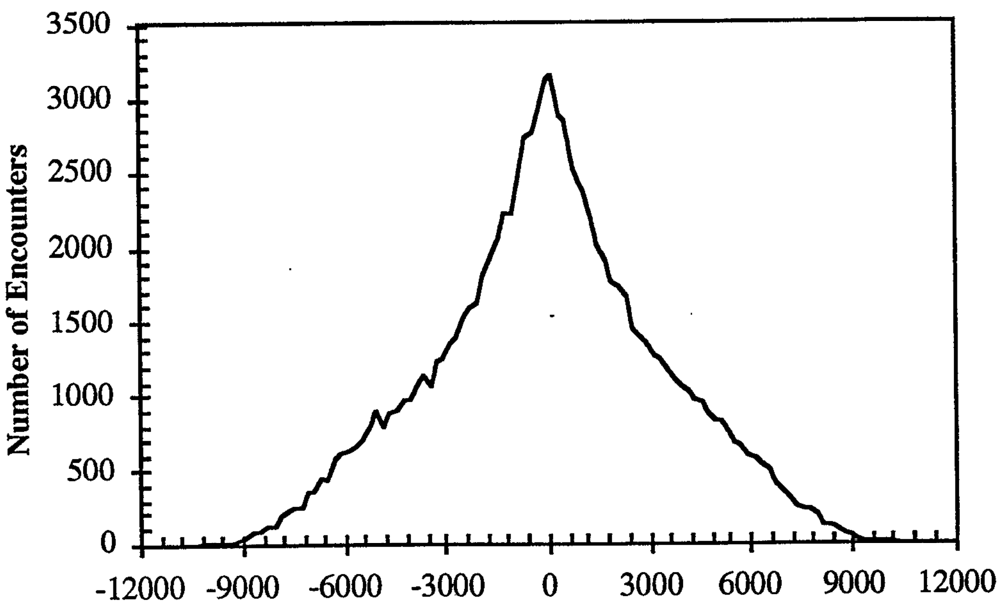

Figure A-4. Miss Distance Histogram - SL5.

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