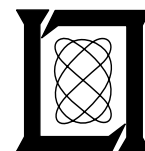


The Influence of Surveillance System Parameters on Automated Conflict Detection and Resolution

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29 November 1972

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16. Abstract The effects of sensor accuracy, data rate, and message delivery delay upon automated conflict detection and resolution processing is analyzed and particular considerations for DABS/IPC operation are discussed. Various options in the design of the algorithmic logic are enumerated and a particular logic is chosen for quantitative inspection. Performance sensitivity calculations for the conflict detection and command generation functions are then presented. The influence of algorithmic logic and traffic environment upon surveillance requirements is delineated.					
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on
Automated Conflict Detection and Resolution

I. INTRODUCTION

In 1968 the Air Traffic Control Advisory Committee recognized the need for a method of reducing the number of mid-air collisions in regions of the airspace containing VFR aircraft. The solution proposed involved "ground-derived collision avoidance instructions data-linked from the ground surveillance equipment to aircraft."¹ This technique was labeled Intermittent Positive Control (IPC). The IPC concept has undergone changes since then, and will undoubtedly change further. However, the system described by ATCAC may be taken as a starting point for the study of automated ground-based conflict detection and resolution systems. The FAA has begun development of a conflict detection technique which would serve as a controller aid. Automatic detection of hazards would alert the controller and suggest corrective actions. The logic for this type of service is very similar to IPC logic except for the fact that the solution generated by the computer is given to the controller as a suggestion rather than being transmitted automatically to the aircraft. Although we shall focus on some particular IPC problems, most of our results will also be applicable to these types of conflict processing systems.

The type and quality of IPC service which can be offered is dependent upon the surveillance and communication systems which are available to support it. For this reason new surveillance systems such as the Discrete Address Beacon System (DABS) must be capable of supporting the types of IPC service which may be implemented during the next couple of decades. The study presented here is intended to accomplish two objectives. First we wish to establish the understanding and methodology necessary for the analysis of IPC performance. Then we wish to determine insofar as possible the relationship between IPC performance and characteristics of the surveillance and communication systems. We begin with a discussion of various IPC concepts and then present some quantitative results for a particular IPC configuration.

II. IPC PERFORMANCE AND SURVEILLANCE SYSTEM INTERACTIONS

A. Performance Parameters

In evaluating IPC performance there are several items which contribute most strongly to the cost/benefit considerations. These items which we will call "performance parameters," are displayed in Table 1.

Table 1. IPC Performance Parameters.

Performance Parameter	Explanation
Safety Level	May be expressed as the system failure rate.
Degree of Control	A measure of the extent to which VFR freedom is eroded by the invoking of positive control. May be expressed in terms of command rate or percentage of time under control.
Induced Pilot Workload	IPC decreases workload in some areas by aiding in collision avoidance, but may require pilot responses which add to work load.
Extent, Availability of Service	Altitude coverage and area coverage are important here.
Cost of Ground Support	Includes data processing capabilities, surveillance and communication facilities, operational personnel, regulatory expenses, etc.
Compatibility	IPC must not interfere with ATC separation services, ATC procedures, tower control, navigation, etc.

Investigations have revealed that two of the most critical performance parameters are degree of control and induced pilot workload. These two performance parameters are closely related in some ways since invoking positive control often requires a response by the pilot. We could propose the simple relationship:

Total workload = rate of negative commands x work factor for negative +
rate of positive commands x work factor for positive.

It seems obvious that negative commands are less bothersome (on the average) than positive, and so the work factor for negative commands is less than that for positive. The work factor would also depend upon particular features of the IPC logic (see discussion in Section IV.)

Several investigators have focused attention on negative commands simply because they are issued more often. But in some cases the work factors are such that positive commands become the more critical factor.

The characteristics of the surveillance and communication systems affect each performance parameter. We will now discuss the aspects of these support systems which are most important to IPC performance.

B. Tracking Accuracy

In determining the degree of hazard which exists we must rely upon tracking data which is subject to various errors. Because of a necessarily conservative approach to hazard evaluation, the presence of tracking errors leads to issuance of commands at times when no true hazard exists. In order to justify non-interference in an encounter, the ATC system must ensure that the miss distance which is obtained from tracking estimates is great enough to accommodate three terms:

- (1) Displacements due to maneuvers which the aircraft might undertake during the immediate future.
- (2) Errors in tracking estimates.
- (3) The required minimum miss distance.

If the contributions of items 1 and 3 are great, then tracking errors may not be of major significance in determining the rate of alarms. However, if the assumed acceleration is limited (through knowledge of flight plan or issuance of restrictive commands), the tracking errors may well be the major factor in the production of alarms.

The tracking accuracy which may be achieved is primarily dependent upon the single-scan position measurement accuracy of the sensor and the rate at which observations are obtained. In addition, information or assumptions concerning aircraft acceleration or speed limits can change our interpretation of collected data and have a significant impact on tracking accuracy.

C. Message Delivery Delay

Uncertainties in the prediction of aircraft position increase as we look further into the future due to the fact that velocity errors integrate over time. Thus, delays which increase the projection or warning times of the system will increase the required rate of alarms.

For a system in which the message delivery depends upon accessing the aircraft via a rotating beam, the rotation time represents the minimum time period between update of the track file and issuance of the command. For such systems, calculations associated with command generation should be completed within the beam rotation period. Note that since we cannot suddenly access the data link for urgent commands, the conflict evaluation logic must anticipate message delivery delays.

D. Coverage

IPC service obviously cannot be extended to regions for which surveillance coverage is lacking. There are also difficulties involved in operating near the coverage boundaries. Resolution is hampered by the fact that aircraft cannot be vectored into uncovered areas. Boundary areas will also contain "pop-up" targets which have not yet completed the entry procedures which establish IPC control. Thus, there will be a buffer zone in which surveillance coverage exists, but effective service cannot be offered. In many areas general aviation aircraft desiring IPC service may find themselves restricted to an altitude band between a lowered positive control boundary (around 10,000 ft) and the beginning of the IPC buffer.

Furthermore, a particular point of traffic concentration (and consequent high collision risk) is found at low altitudes near uncontrolled airports. Clearly, surveillance coverage to the lowest possible altitude is desirable from an IPC viewpoint.

E. Message Display

It is essential that a display unit be available which presents the required IPC information in an unambiguous and easily interpreted format. The most useful unit would be capable of displaying either traffic advisories or IPC commands. It would be capable of displaying several traffic alarms simultaneously and issuing audible signals to the pilot when the display changes in a significant way.

F. Interrogation Management

In IPC operation, the tracking accuracy which is needed varies over short periods of time according to the stage of the avoidance process in which we find the aircraft. For instance, if an aircraft happens to be well separated from other aircraft, the quality of information required conforms to that needed for the initial hazard detection. The greatest accuracy is needed when the system is in the process of generating resolution instructions. Many sensor configurations have the ability to vary the mode of tracking either by employing different data rates for different aircraft, or by using information from more than one sensor. With phased array DABS operation, one may take the view that in each time period, a certain number of interrogations are permitted. These interrogations may be divided up among aircraft according to need. The result is that the effective data rate (in terms of system performance) can be made larger than the average data rate.

III. CALCULATION OF CONFLICT RATES

A. Encounter Rate Models

In the discussion of IPC performance in Section II, we saw that the rate of IPC commands is closely related to the critical performance parameters. Because tracking accuracy is important in determining the necessary rate of commands, we will investigate this aspect of IPC operation in more detail. We begin by describing a well known method of calculating conflict rates.

The frequency with which collisions arise can be estimated through use of a model commonly employed in the study of gasses. In developing this model we first consider a case in which the aircraft of interest has velocity \vec{V}_1 and all other aircraft have velocity \vec{V}_2 . The geometry of an encounter is indicated in Figure 3-1. As long as the aircraft velocities remain constant, the pilot of aircraft 1 observes all aircraft of velocity \vec{V}_2 passing him with relative velocity $\vec{V} = \vec{V}_2 - \vec{V}_1$. The major parameters defining the trajectories are the relative velocity, \vec{V} , and the miss distance, D . The frequency with which aircraft of the second type come within a distance D of the first aircraft is given by

$$\lambda = 2 D \rho V \quad (3-1)$$

where ρ is the area density of the aircraft and V is the magnitude of the relative velocity. Suppose now that aircraft exist with various velocities so that $f_V(x)$ is the probability density function for relative velocity $V = x$. The contribution to the encounter rate due to aircraft in dx at x is then

$$d\lambda = 2 D \rho_0 f_V(x) dx \quad (3-2)$$

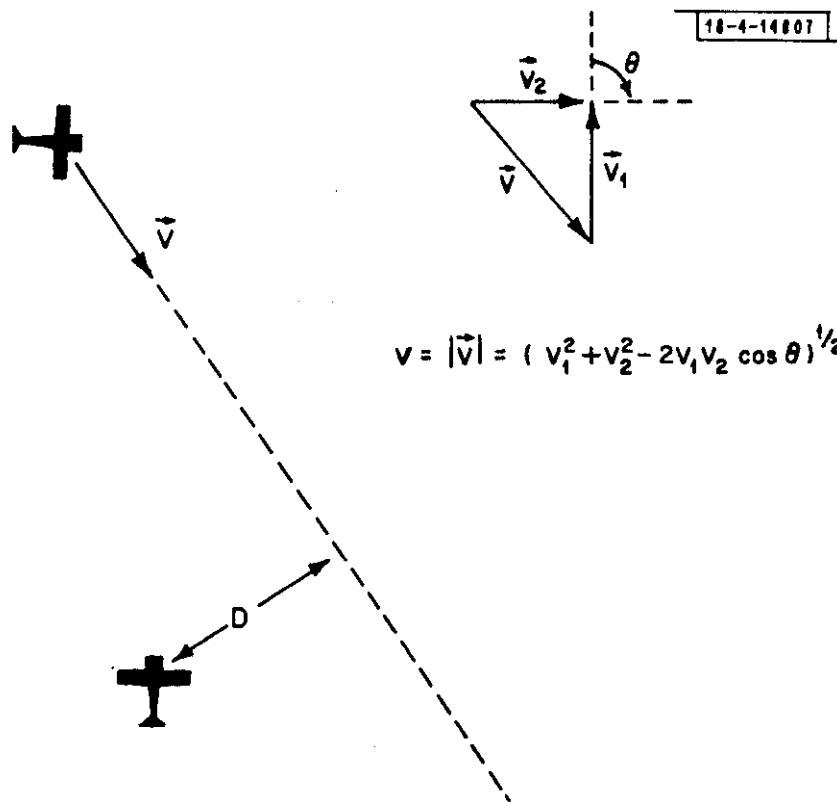


Figure 3-1. Geometry of an Encounter Between Two Aircraft.
 (θ = Encounter Angle, \vec{V} = Relative Velocity,
 D = Miss Distance.)

where ρ_0 is the area density for all aircraft and $\rho_0 f_V(x) dx$ is the area density for aircraft producing a relative velocity in the interval dx at x . Since the total encounter rate is the sum of the encounter rates at each velocity, we integrate equation (3-2) to obtain

$$\lambda = E [2 D \rho_0 V] \quad (3-3)$$

Note that if there are N aircraft in the system, the detection algorithm will observe encounters at a rate $N\lambda/2$.

If all aircraft fly at speed V_0 with totally random headings, the expected value of V is

$$E[V] = \frac{4}{\pi} V_0 \quad (3-4)$$

This results in the "gas model" expression

$$\lambda = \frac{8}{\pi} \rho_0 D V_0 \quad (3-5)$$

B. Nonuniform Heading Distribution

Investigation has shown that the result of the above approach is highly sensitive to the assumed distribution of headings. The gas model assumes headings uniformly distributed over 2π . In reality, several factors serve to produce nonuniform distributions of aircraft headings. In any given region, there are predominant directions of travel which usually correspond to the

paths connecting major population centers. In the Los Angeles basin, for example, it has been estimated that about 65% of the flights are in a north-south direction. At a given altitude, headings are also affected by the cruise altitude rules (FAR 91.109, 91.121) which specify that aircraft with easterly courses (0° to 179°) fly at odd thousands plus 500 feet (i.e., 5500', 7500', etc.) and aircraft with westernly courses (180° to 359°) fly at even thousands plus 500 feet. This means that at a given altitude we may find most aircraft within a heading interval of width 180° . A third factor in heading determination is the nature of the VOR navigation system which encourages pilots to fly radials to or from VOR locations. Radial flying reduces velocity differences in most of the airspace (but results in increased traffic density over the VOR site).

Let us consider a case in which aircraft headings are uniformly distributed over some interval $[0, \theta_L]$ where $\theta_L \leq \pi$. Now the heading difference is no longer uniformly distributed but follows a density function

$$f(\xi) = \frac{2}{\theta_L} (\theta_L - \xi) \quad \xi \leq \theta_L \quad (3-6)$$

We thus find

$$E[V] = E[|\vec{V}_1 - \vec{V}_2|] = \frac{8V}{\theta_L} \left(1 - \frac{2}{\theta_L} \sin \frac{\theta_L}{2}\right) \quad \theta_L \leq \pi$$

and thus equation (3-5) is corrected to read

$$\lambda = \frac{16}{\theta_L} \rho_0 D V_0 \left(1 - \frac{2}{\theta_L} \sin \frac{\theta_L}{2}\right) \quad (3-7)$$

The reduction in the number of conflicts is shown in Figure 3-2 where we plot the ratio ν of the conflict rate for headings in θ_L versus the conflict rate for headings distributed over 2π (gas model). It is evident that a considerable reduction in the rate of conflicts can be achieved if airspace rules encourage aircraft with similar headings to fly at the same flight levels. The current cruise altitude rules can yield a 27% reduction in the conflict rate. If the altitude-heading relationship were carefully chosen to take advantage of the traffic patterns of a given region, greater reductions could be expected.

C. Application to Command Rate Determination

The above equations can be applied to the calculation of the command rate for IPC systems. Because most encounters take place between aircraft on rectilinear flight paths, we expect the command rate under the condition of universal rectilinear flight to closely approximate the actual command rate. Suppose that for a given rectilinear geometry described by relative velocity V and miss distance D , we determine that there is a probability p of command issuance. The rate of commands is related to the product of the encounter rate and the probability of commands at each encounter. Thus,

$$\lambda(\text{commands}) = E [2pD \rho_0 V] \quad .$$

In this expression, the only factor which is directly dependent upon the surveillance system is p . For a given miss distance, the presence of surveillance errors increases the likelihood that a command will be issued. We may

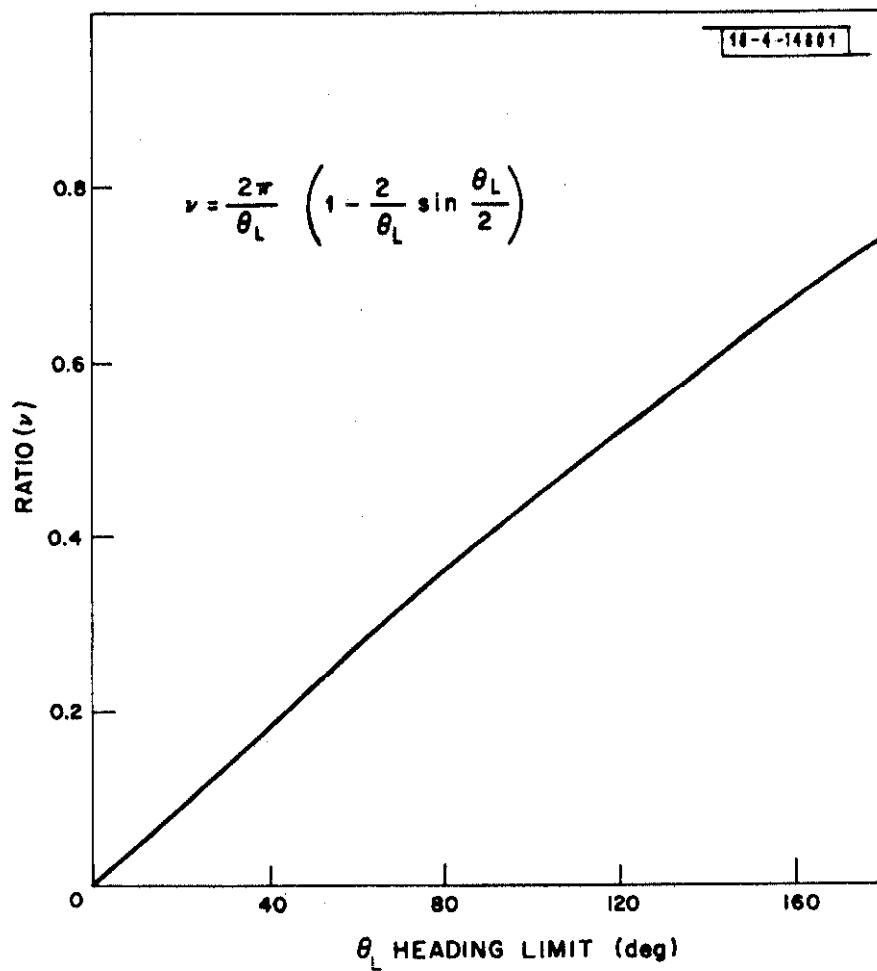


Figure 3-2. Encounter Rate for Headings Between 0 and θ_L Given as Fraction of the Gas Model Value.

also note that when D is small, p approaches unity (because very small miss distances almost certainly trigger commands). When D is large, p approaches zero (since it is unlikely that an alarm will occur when the aircraft do not pass close to each other).

We shall now define a characteristic value of the miss distance which we shall call the command cross section. Its value is such that for a given encounter geometry, the actual rate of commands is the same as if all passages within this distance result in commands while passages outside this distance never result in commands. As we shall see later, the value of the command cross section is different for different encounter geometries. We can compare two IPC designs "pointwise" by comparing their cross sections for particular encounter situations. To compare them in an overall sense, we must have a traffic model which specifies the frequency with which each encounter geometry occurs. A weighted average of the cross sections can then be calculated.

D. Multiple Conflicts

At lower traffic densities, simultaneous conflicts between three or more aircraft are rare compared to conflicts between aircraft pairs. But, as densities increase, a higher incidence of multiple conflicts will be observed. Such conflicts are inherently more difficult to resolve than simple pair encounters, and will often force the IPC system to issue less efficient, more restrictive commands than would be required for isolated encounters.

Some idea of the frequency of multiple conflicts can be obtained from considering a simple model in which encounters occur at rate λ , each producing

a conflict of duration τ . According to the resulting Poisson distribution, the probability that k encounters occur in the time interval τ is

$$P[N = k] = \frac{(\lambda\tau)^k e^{-\lambda\tau}}{k!} .$$

The probability that at least one additional conflict will arise during the interval τ is $1 - P[N = 0] = 1 - e^{-\lambda\tau}$. This expression is evaluated in Table 2.

Table 2. Fractional Incidence of Multiple Conflicts.

Rate . Duration = $\lambda\tau$	Fraction of Multiple Conflicts = $1 - e^{-\lambda\tau}$
0.10	0.096
0.25	0.222
0.50	0.394
0.75	0.528
0.90	0.594

IPC systems may operate with λ as high as 60/hour. For $\tau=15$ seconds we obtain $\lambda\tau = 0.25$, indicating that 22% of all conflicts will involve more than two aircraft. This fraction indicates that for IPC operation with high conflict rates, multiple conflicts may be significant.

IV. FEATURES OF IPC DESIGN

In this section, we shall discuss certain features of the IPC logic which influence system performance. We will try to give the reader an idea of the numerous options available in the design of the decision making logic for the system. In many cases the most desirable option cannot be determined without operational experience. In choosing a particular structure for analysis, we do not expect to obtain a system which fully meets the complex criteria that would be applied to it in the real world. We do hope to obtain an approximate representation of those aspects of IPC operation which are of most interest to us.

During the ATCAC study it was recognized by Willis et al that unless proper consideration is given to the structure of the IPC hazard evaluation algorithm, the required data processing load can be enormous. The primary difficulty arises from the fact that N aircraft produce $N(N-1)/2$ possible conflicting aircraft pairs and we do not wish to issue a command to any one pair without subjecting it to a fairly sophisticated hazard analysis. The solution to the problem is now well known: An initial sorting procedure using rather crude but computationally efficient techniques serves to identify that small fraction of aircraft pairs whose proximity to each other indicates a possible hazard. To these pairs only, we then apply more sophisticated hazard criteria. At each stage the number of aircraft which are considered to be in hazard decreases until we are left with a relatively small number to which our most sophisticated evaluation is applied. Studies of computational aspects for algorithms of this type² have indicated that data processing loads are acceptable. In our analysis we are concerned mainly with command

generation and we have addressed ourselves to the more sophisticated evaluation criteria which might be employed in the final stages of the IPC logic.

A. Negative and Positive Commands

In the ATCAC concept there are two types of IPC instructions: positive (Do) commands and negative (Don't) commands. The IPC logic must specify the criteria for issuance of each type of command. Negative commands may prohibit maneuvers in a specific direction (Don't turn left) or in all directions (Don't turn). Specific commands are less restrictive in that they allow the pilot the option of turning in the direction which has not been prohibited. However, as one aircraft crosses the path of another it is sometimes necessary to change a "Don't turn left" command to "Don't turn right." Between these two conditions we may need a period in which a "Don't turn" command is in effect.

One can imagine a quite safe IPC strategy which utilizes positive commands only. But negative commands do serve useful purposes. Among these are:

- (1) They allow the system to prevent the development of more severe hazards in a way that requires the minimum amount of intrusion.
- (2) They alert the pilot to hazardous situations before he must begin his avoidance maneuver. This allows the pilot to evaluate the situation when factors which we have not accounted for (such as cloud proximity or aircraft malfunctions) conspire to nullify IPC effectiveness.
- (3) They produce a period of reduced acceleration potential which can be used to improve tracking accuracy before positive commands are issued.

- (4) They allow us to give a positive command to only one aircraft and be confident that a maneuver by the other aircraft will not cancel its intended effect.

Additional comment is needed upon Point 3 above. Tracking algorithms utilize data taken over an extended period in the past in order to produce the best possible estimates of current trajectory parameters. For the type of surveillance system considered here, only position measures are available. In most instances the aircraft is flying straight (zero accelerations). Because the velocity is constant, all data collected during straight line flight is useful in determining the trajectory parameters. However, if the aircraft suddenly begins to turn, the fact that we are combining measurements from the straight line portion of the trajectory with measurements from a curved portion leads to dynamic errors, or "biases." The estimate of current velocity will be biased in the direction of the previous velocity and the estimate of position will be biased toward the position the aircraft would have if it had not turned. The total error which the tracking algorithm must seek to minimize is a combination of bias error and measurement noise error.

The most common way of allowing for unknown accelerations is to adjust the tracker parameters so that only the more recent measurements have a significant effect on the output. This reduces the bias error, but since the tracker is effectively employing a reduced data base, the contribution of the noise errors increases. Knowledge concerning the aircraft acceleration allows us to set our tracker parameters in an optimum fashion and thus reduce the total error. For this reason a period of known rectilinear (zero acceleration) flight is useful.

As these considerations imply, the issuance of negative commands tends to decrease the required number of positive commands. This fact seems to be of primary importance in choosing parameters for negative command issuance. If we decide that positive commands are much more troublesome to the pilot than negative commands, we will choose to operate with a high ratio of negative-to-positive commands.

B. Lateral Resolution

1. Turn magnitude

Of considerable interest is the magnitude of the collision avoidance maneuvers. We could establish standard maneuvers in which case "Do turn right" would mean that the aircraft should change its heading a fixed number of degrees to the right. A better approach is to provide the pilot with the heading to which he should turn. The command would then be of the form "Do turn right to 150 degrees." With this approach we may require small turns early in the encounter or large turns late in the encounter. In general the use of large turn magnitudes decreases the required number of positive commands while producing turns which are more disruptive when they do occur. The small heading changes must remain in effect for longer periods of time and thus increase the percentage of the flight time that the aircraft is under positive control.

2. Priorities, right-of-way rules

The decision concerning which aircraft will receive the command may be based on assigned priorities, fixed rules, or resolution efficiency.

Assigned priorities will be necessary when airborne emergencies arise, and may also be given to aircraft which are known to be involved in critical navigational operations.

Fixed right-of-way rules exist (FAR 91.67) which attempt to define the type of resolution which should be carried out in VFR-VFR encounters. The regulations state in part that "when aircraft of the same category are converging at approximately the same altitude (except head-on, or nearly so) the aircraft to the other's right has the right-of-way; he shall give way to that aircraft and may not pass over, under, or ahead of it, unless well clear." It would be attractive if we could utilize this set of rules in the IPC system in order to allow the pilot to anticipate or confirm IPC instructions from his own observations. Difficulties arise, however, on the following points:

- (1) The pilot and the ground may disagree on whether or not a passage is "well clear." They may also come to different conclusions concerning whether or not the encounter is "head-on or nearly so."
- (2) The other aircraft may possess the right-of-way by virtue of an assigned priority. In this case, the geometry of the encounter would have no bearing on right-of-way.
- (3) In cases where multiple aircraft are involved, the magnitude of the turn as well as the direction may be important. Pilot-initiated turns of too great a magnitude may interfere with resolution, even when the turn itself is in the right direction.
- (4) Fixed rules will occasionally force the least efficient resolution maneuvers to be chosen.

3. Direction of turn

Consider an encounter as depicted in Figure 4-1 in which aircraft A is to receive the positive command. There are two possible directions for lateral maneuvers. A right turn corresponds to a turn parallel strategy in which A turns parallel to the path of B. A left turn corresponds to a turn opposite strategy which results in A turning anti-parallel to B. The turn opposite strategy results in an increased relative velocity which shortens the duration of the encounter. The turn-parallel strategy reduces the relative velocity and as a result aircraft A may fly far from its intended course before it is finally clear of aircraft B. One undesirable result of this may be a recurrent conflict as indicated in Figure 4-2.

Once we determine the set of maneuvers which are allowed by the IPC procedures, the logic should choose from that set the maneuver which is most efficient in resolving the conflict. In some cases this means choosing the maneuver which takes advantage of the miss distance which already exists. In other cases the directions of maximum aircraft acceleration is the deciding factor. The decision-making algorithm can employ the straightforward approach of projecting the trajectories ahead under conditions corresponding to each allowable command set. The relevant parameters (miss distance, time under control, etc.) are then tabulated and the best maneuver chosen.

All we have done here is indicate some of the considerations which are involved in the choice of the command to be issued. There are several pilot acceptance problems which cannot be resolved without experiment (such as whether or not the maneuver appears safe, whether loss of visibility due to aircraft banking is significant, etc.).

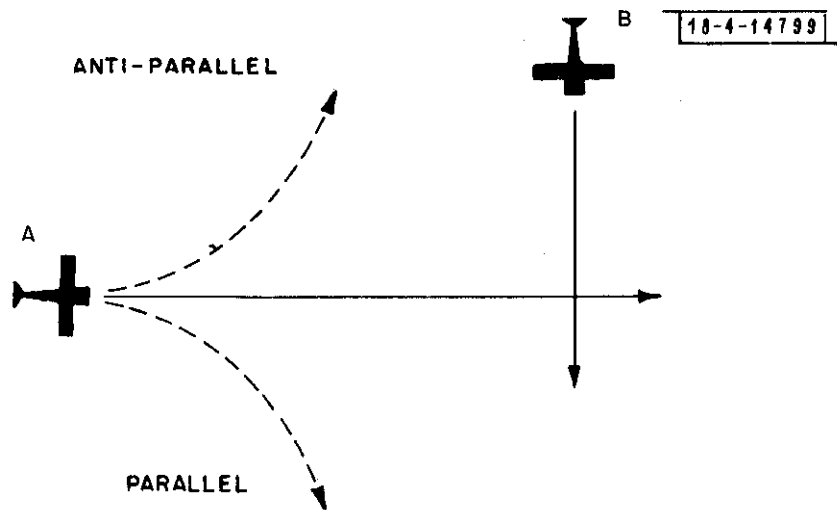


Figure 4-1. The Choice of Turn Direction.

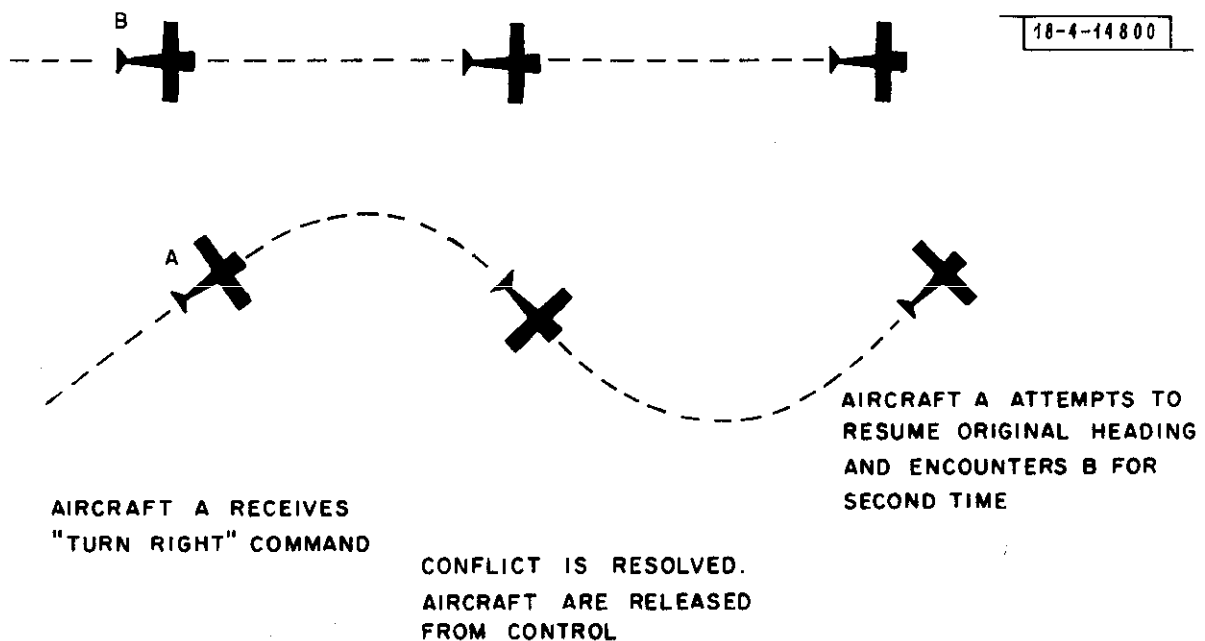


Figure 4-2. Example of How a Conflict May Recur.

C. Resolution in altitude

Encounters between aircraft occur in three-dimensional space and IPC logic must eventually incorporate the resulting geometrical considerations. A collision is possible only if insufficient separation occurs simultaneously in the lateral and vertical directions. Because of the differences in flight dynamics and error inputs, it is expedient to employ separate techniques for evaluating the conflict in each plane.

We shall examine a set of warning criteria for the vertical dimension. Changes in vertical velocity will be represented by step-function changes with suitable delays and we shall issue negative commands to both aircraft simultaneously. Consider first the case in which no commands have yet been issued. The worst case separation at time t into the future is calculated under the assumption that the command process is initiated at the current evaluation time. Then

$$H(t) = \hat{H}_0 - H_e + \dot{H}_0 t_d + \dot{H}_1 t_d + \dot{H}_2 (t - 2t_d) \quad t \geq 2t_d \quad (4-1)$$

where

$H(t)$ is the altitude separation at time t

\hat{H}_0 is the current reported altitude separation

H_e is the worst-case error in \hat{H}_0

\dot{H}_0 is the worst-case separation rate when no commands have been issued

\dot{H}_1 is the worst-case separation rate when negative commands are in effect

H_2 is the worst-case separation rate when positive commands are in effect

t_d is the delay time.

We must ensure that the aircraft achieve a safe vertical separation by the time their approach in the lateral plane becomes hazardous. The required warning time $t_w(-)$ is found from equation (4-1) by setting $H(t) = H_{req}$ (where H_{req} is the required separation) and solving for t . Thus,

$$t_w(-) = 2t_d + \frac{H_{req} - \hat{H}_0 + H_e - \dot{H}_0 t_d - \dot{H}_1 t_d}{\dot{H}_2} \quad (4-2)$$

In the case of positive command, we assume that negative commands are already in effect, so that

$$H(t) = \hat{H}_0 - H_e + \dot{H}_1 t_d + \dot{H}_2 (t - t_d) \quad t > t_d$$

and the resultant warning time is

$$t_w(+) = t_d + \frac{H_{req} - \hat{H}_0 + H_e - \dot{H}_1 t_d}{\dot{H}_2}$$

These expressions are evaluated in Figure 4-3 for the parameter values

$$\begin{aligned} t_d &= 6 \text{ seconds} \\ \dot{H}_0 &= 1800 \text{ fpm} = 30 \text{ fps} \\ \dot{H}_1 &= 60 \text{ fpm} = 1 \text{ fps} \end{aligned}$$

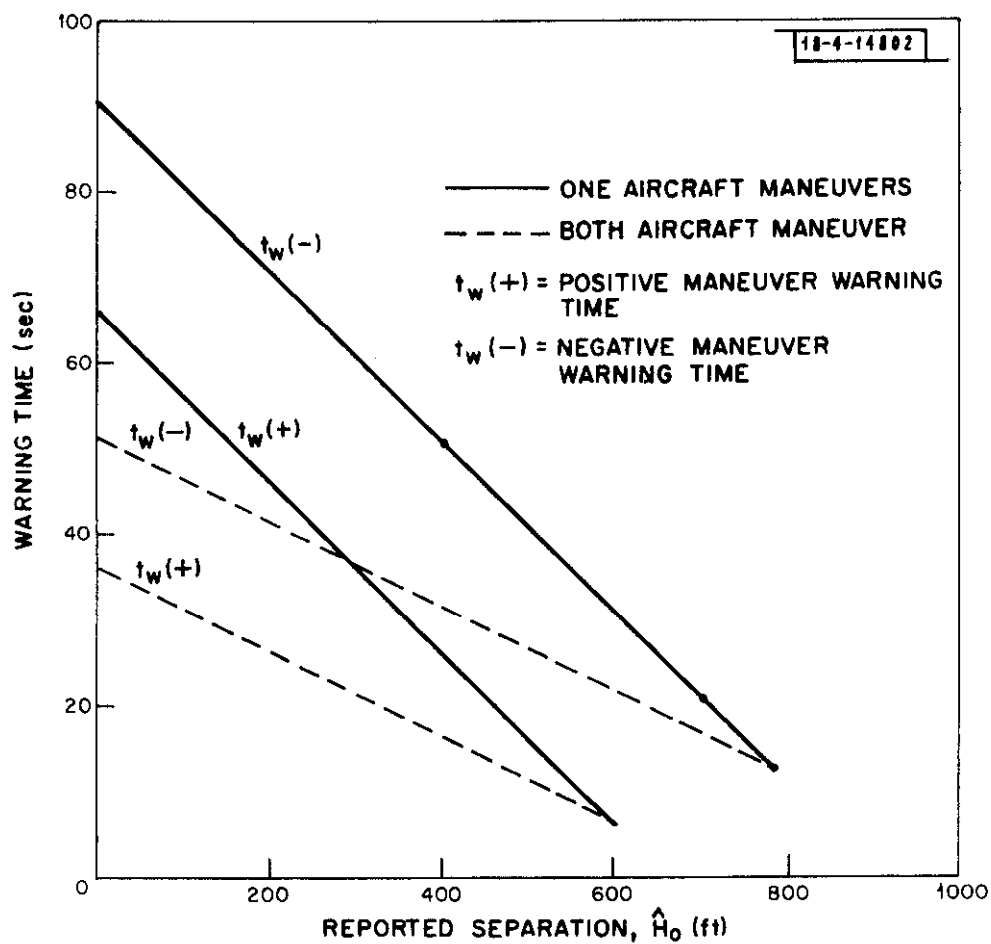


Figure 4-3. Warning Time Required to Ensure Resolution for Vertical Maneuvers.

$$\begin{aligned}\dot{H}_2 &= 600 \text{ fpm} = 10 \text{ fps if only one receives positive command} \\ &= 1200 \text{ fpm} = 20 \text{ fps if both receive positive command} \\ H_e &= 400 \text{ ft} \\ H_{\text{req}} &= 200 \text{ ft.}\end{aligned}$$

In general, we do not want to use vertical maneuvers when they require a greater warning time than lateral maneuvers. To do so would mean projecting our track ahead over a longer period of time, thus incurring larger prediction uncertainties and more false alarms. A typical warning time for lateral resolution is 30 seconds. From inspection of Figure 4-3 it is obvious that lateral resolution is more efficient when the aircraft are reported at the same altitude. But, for the $t_w(+)$ one-maneuver curve, we see that altitude separations from 360 to 606 ft allow warning times less than 30 seconds. In this case, assuming uniform altitude distribution, vertical resolution would possess a warning time advantage in about 40% of the encounters. Although other factors will influence our choice of maneuvers, it does seem that vertical resolution will be preferred in some fraction of the encounters.

The proper integration of vertical and lateral resolution algorithms is beyond the scope of this report. We will examine lateral resolution in a maneuver which does not incorporate the possibility of vertical resolution. The cross section which we obtain will correspond to the maximum lateral width of the alarm region.

V. RESULTS FOR A PARTICULAR IPC ALGORITHM

A. Algorithm Design Choices

In this section, we develop a particular command issuance algorithm for both negative and positive commands and determine the effect of tracking errors on the associated command rate. The design choices identified earlier will be held fixed while we vary tracking accuracy. The features of the design we will employ include the following:

- (1) Negative commands are issued to both aircraft simultaneously.
- (2) In the issuance of positive commands, we shall attempt to resolve each hazard by giving commands to only one aircraft of the pair.
- (3) Positive commands require the aircraft to execute large-magnitude heading changes.
- (4) The decision as to which aircraft should receive the positive command is based upon resolution efficiency.
- (5) When the heading difference is less than or equal to 90° , we shall employ anti-parallel turns rather than parallel turns.

B. Negative Command Algorithm

In Section IV.A. we listed several benefits which accrue from the issuance of negative commands. In some cases negative commands are merely preparatory to the actual resolution of the hazard by positive commands. Because of varied considerations involved, it is difficult to decide exactly how early negative commands should be issued. If we desired to reduce the rate of negative commands to a minimum, we would delay issuance almost until the point when positive commands become necessary. But, we recall that the earlier the negative

command is issued, the less likely is the eventual need for a positive command. Therefore, the optimum "warning time" for negative command issuance is affected by the trade-off between negative and positive command rates. Table 3 lists the sequence of events which occur during resolution.

Table 3. Sequence of Events in Conflict Resolution.

TIME	EVENT
$t_0 =$	Interrogation provides observation of aircraft
$t_1 = t_0 + \text{data gathering period}$	Begin conflict filtering
$t_2 = t_1 + \text{computation time}$	Decision made that negative commands needed
$t_3 = t_2 + \text{command link access time}$	Negative commands sent to both aircraft
$t_4 = t_3 + \text{pilot/aircraft response time}$	Aircraft in compliance with negative commands
$t_5 = t_4 + \text{intermediate tracking interval}$	Need for positive command determined
$t_6 = t_5 + \text{command link access time}$	Positive command sent to selected aircraft
$t_7 = t_6 + \text{pilot/aircraft response time}$	Aircraft begins to execute avoidance maneuver
t_8	Closest approach occurs
Note: If we ask the pilot to verify his acceptance of the IPC command, the time required for this process must be added to the above sequence.	

The minimum warning time is constrained by the time required to complete this sequence. Warning times as low as 20 seconds are possible for some IPC systems, but communication delays and conservative assumptions concerning pilot response can push the minimum warning time above 40 seconds.

Another parameter which must be chosen is the magnitude of aircraft maneuvers which we anticipate. The more severe the anticipated acceleration we must guard against, the more frequently will negative commands be issued. It seems that any IPC system must perform well against the normal range of aircraft maneuvers, but it may be possible to prohibit severe or acrobatic maneuvers for aircraft under IPC control.

The calculations we shall use proceed roughly as follows: The estimated trajectories are projected ahead through a time t_w . If the worst-case accelerations and tracking errors (3 sigma) can produce an approach to within 1000 feet at the projected time, we issue negative commands to both aircraft. A simple curve-fitting tracker as described in Appendix A was employed to generate the performance curves of Figure 5-1.

Curves D and E give the collision cross section for cases in which aircraft trajectories are projected ahead with zero turn rates. Curves A, B, and C apply to cases in which we allow worst-case turns at rates up to $3^\circ/\text{sec}$. The added cost in terms of command cross section for protection against maneuvers is evident. For these calculations, we have assumed a one-second data rate and a 6-second delay between track update and initiation of the chosen maneuver. Curves D and E apply to all encounter angles, but curves A, B, and C were generated with $\theta=90^\circ$. In calculating the bias error, an undetected turn corresponding to a $1.5^\circ/\text{second}$ rate is allowed for.

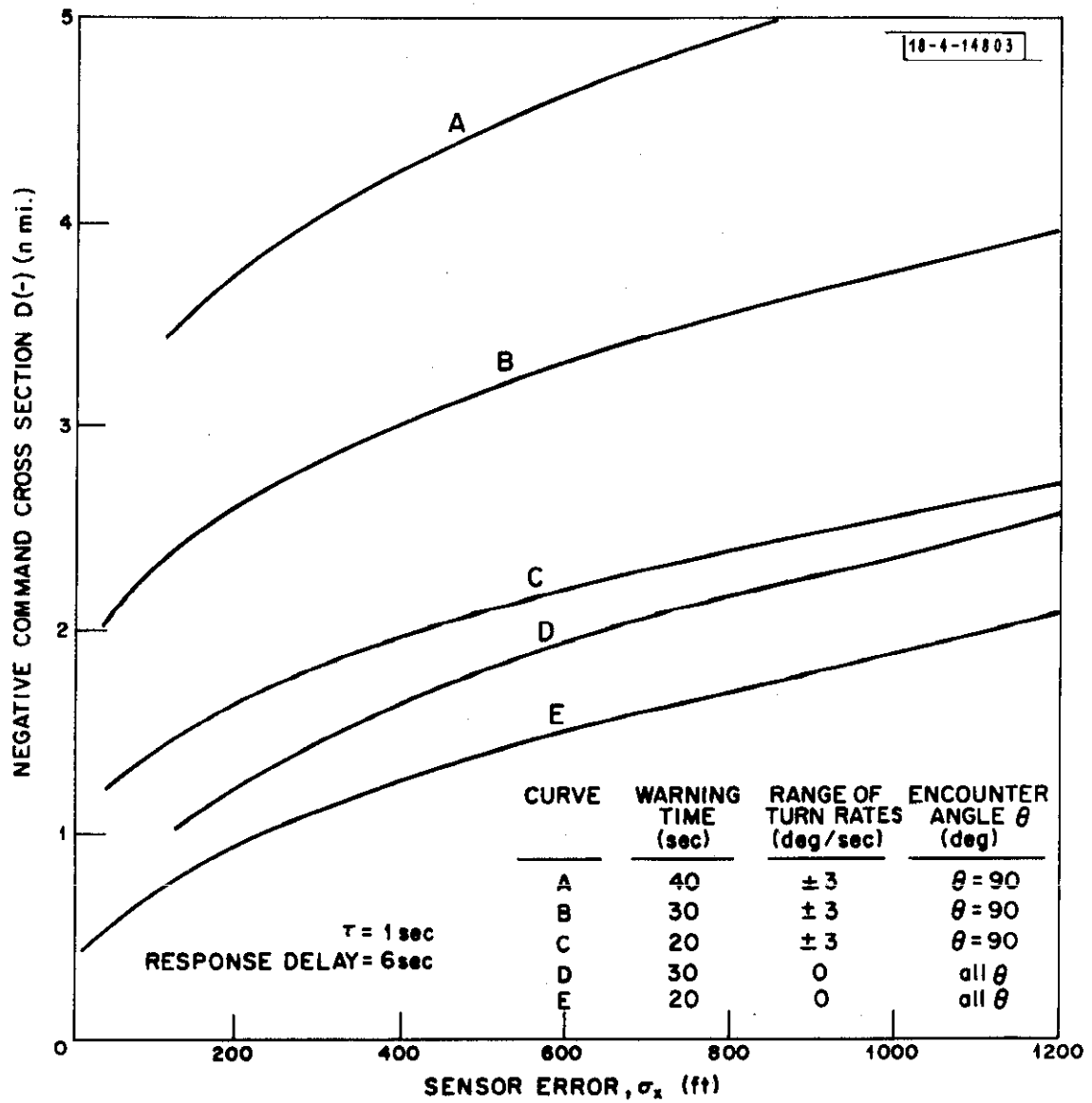


Figure 5-1. Command Cross Sections for Negative Commands.

We can see that for most values of c_x , the curves have a slope $\frac{\partial D(-)}{\partial \sigma_x} \approx 0.10 \text{ nmi}/100 \text{ ft in } \sigma_x$. The effect on the command rate of changes in σ_x can be obtained by evaluating $\frac{\partial \lambda(-)}{\partial \sigma_x} = \frac{\lambda(-)}{D(-)} \frac{\partial D(-)}{\partial \sigma_x}$ using the expression for $\lambda(-)$ in Eq (3-7). It is easily verified that the fractional change in the command rate is

$$\frac{1}{\lambda(-)} \frac{\partial \lambda(-)}{\partial \sigma_x} = \frac{1}{D(-)} \frac{\partial D(-)}{\partial \sigma_x} = \frac{0.10 \text{ nmi}}{D(-)} \text{ per 100 ft in } \sigma_x.$$

Thus, if $D(-) = 2.0 \text{ nmi}$, a 5% change in the negative command rate is observed for each 100 ft degradation in sensor accuracy.

The sensitivity to warning time can be estimated through comparison of curves A, B, and C. We obtain

$$\frac{\partial D(-)}{\partial t_w} \approx 0.13 \text{ nmi/second of warning time}$$

If the sensitivity here is compared with the results of the preceeding paragraph, we see that an increase of 0.83 second in warning time has approximately the same impact as a 100 ft increase in sensor accuracy.

The change in tracking parameters which accompanies the issuance of negative commands affects all succeeding calculations. In the positive command calculations which are made in later sections, we shall choose negative issuance times which correspond to curve D of Figure 5-1. These issuance times are displayed in Figure 5-2. Note that as the sensor error increases, command issuance must occur at earlier times to ensure safety.

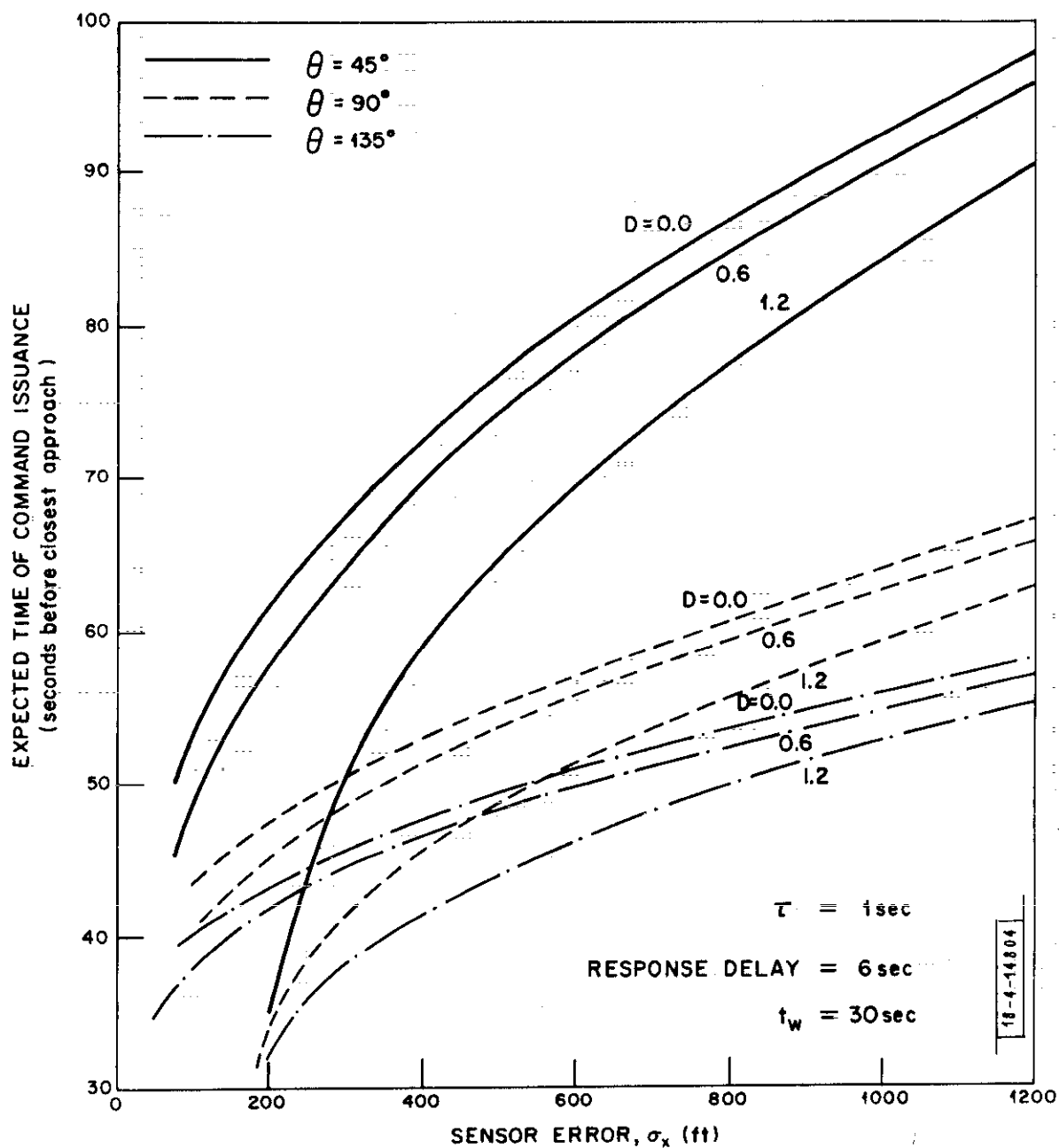


Figure 5-2. Expected Time Before Closest Approach at Which Negative Commands are Issued.

C. Positive Command Algorithm

1. Resolution strategy

We shall now discuss the algorithm employed in the issuance of positive commands. According to the logic previously described, negative commands are first issued to both aircraft. Each time the track file is updated, we will reevaluate the situation to see if a positive command is required. We desire an algorithm which always provides commands with sufficient warning time in truly hazardous circumstances, but is not overcautious to the point of issuing an excessive number of commands. Note that there is an advantage in waiting as long as possible before issuing the command since updated track information may reveal that no positive commands are necessary. Two factors which are important in this respect are:

- (1) Improving track accuracy due to the change in tracker parameters which is allowed when negative commands are in effect, and
- (2) A time to closest approach which is decreasing, resulting in a shorter prediction interval and decreased prediction errors.

A particular resolution command will be expressed in notational form (X_A, X_B) where X_A is the command given to aircraft A, and X_B is the command given to aircraft B. The commands have five possible values as indicated below:

- X=0 Negative Command Only
- X=L Turn Left
- X=R Turn Right
- X=C Climb
- X=D Descend

After negative commands are issued, the set (0,0) is in effect. The command sets which we shall consider for lateral resolution are (L,0), (R,0), (0,L) and (0,R). Command sets requiring both aircraft to maneuver will be used for backup.

When the aircraft are far away from each other, there are several command sets which result in successful avoidance. As the distance between the aircraft decreases, the number of successful options dwindles. Operating under these conditions we might proceed as follows: From the possible command sets we exclude those which are undesirable (such as descent maneuvers for aircraft near lower boundary, maneuvers for IFR aircraft, etc.). At each evaluation time, we examine the remaining options to see if, assuming action is deferred until the next evaluation time, a viable option will exist. When we see that the last successful option is about to disappear, we implement it immediately. Errors and uncertainties are allowed for in the decision making process in order to ensure that the command, when given, will be successful. Note that in this approach, there is no fixed warning time for positive commands. In effect, the needed warning is determined individually for each particular situation.

IPC algorithms employed in previous investigations have used one set of criteria for deciding when to issue commands and then used an independent set of criteria for selecting the specific command set to be issued. This type of algorithmic structure cannot easily relate the effectiveness of the maneuver to the need for its issuance. As a result, there is a tendency to either issue the command too early, or to wait until it is too late to issue commands with certain desirable features. The procedure we suggest here overcomes these tendencies by first specifying the desirable command sets and then evaluating need-for-issuance under the condition that one of the desirable command sets be employed.

Note that our approach can be modified so that we ask for two viable options and allow the pilots the privilege of rejecting one of them. This type of strategy incurs communication delays (see Section II-C on the effect of delays on IPC performance) and can create problems when one pilot executes the commands while the other rejects them. Further study is needed here. For our current purposes, we shall not include pilot rejection of commands.

2. Resolution calculations

We shall now describe the calculations involved in determining whether or not a given command set will be successful. We first consider the worst-case sequence of events which could result if action were deferred until the next evaluation time:

No command issued at current time.

Next update received, calculation begun.

Calculation shows that command cannot be deferred.

Data link accessed; command sent.

Pilots respond.

Closest approach D achieved at time T .

Let us propose a minimum required miss distance D_0 . For a particular command set, we may calculate D above and compare it with D_0 . If there were no errors in our projection of events, our criteria for success would be $D > D_0$. Suppose, however, that the errors in projecting ahead to time T are characterized by a standard deviation $\sigma(T)$. We might then ask that D satisfy the criteria $D > D_0 + 3\sigma(T)$, i.e., we ask for a confidence interval corresponding to three standard deviations of D . These parameters can be adjusted as desired to give any level of safety or any average miss distance.

The cross section for positive commands, $D(+)$, is presented in Figure 5-3 for several encounter angles. The solid curves were obtained under the assumption that once negative commands are in effect, the aircraft acceleration potential is reduced from an initial value of 0.25 g (turn rate 1.5°/sec) down to 0.033 g (turn rate 0.2°/sec). The dashed curve assumes that the acceleration potential remains at the higher value. By comparing the solid and dashed curves we see that the assumed aircraft acceleration potential has a significant effect on the rate of positive commands. Inspection of these curves also reveals an increase of 6-12% in cross section for each 100 ft degradation of sensor accuracy. Thus, the rate of positive commands is more sensitive to sensor accuracy than the rate of negative commands.

D. Effect of Data Interval

The effect of varying data intervals on the cross section $D(-)$ is displayed in Figure 5-4. Note that essentially constant performance is achieved for constant values of the parameter $\sigma_x \tau^{1/2}$. A similar relationship can be observed for the positive command cross section. This relationship can be explained by inspection of the tracking equations of Appendix A in which tracking errors are proportional to $\sigma_x \tau^{1/2}$. The figures presented in this section can be adapted for various values of the data interval through a scale factor $\tau^{1/2}$. For instance, a sensor with $\sigma_x=600$ ft, $\tau=1$ sec should offer about the same performance as one with $\sigma_x=300$ ft, $\tau=4$ sec.

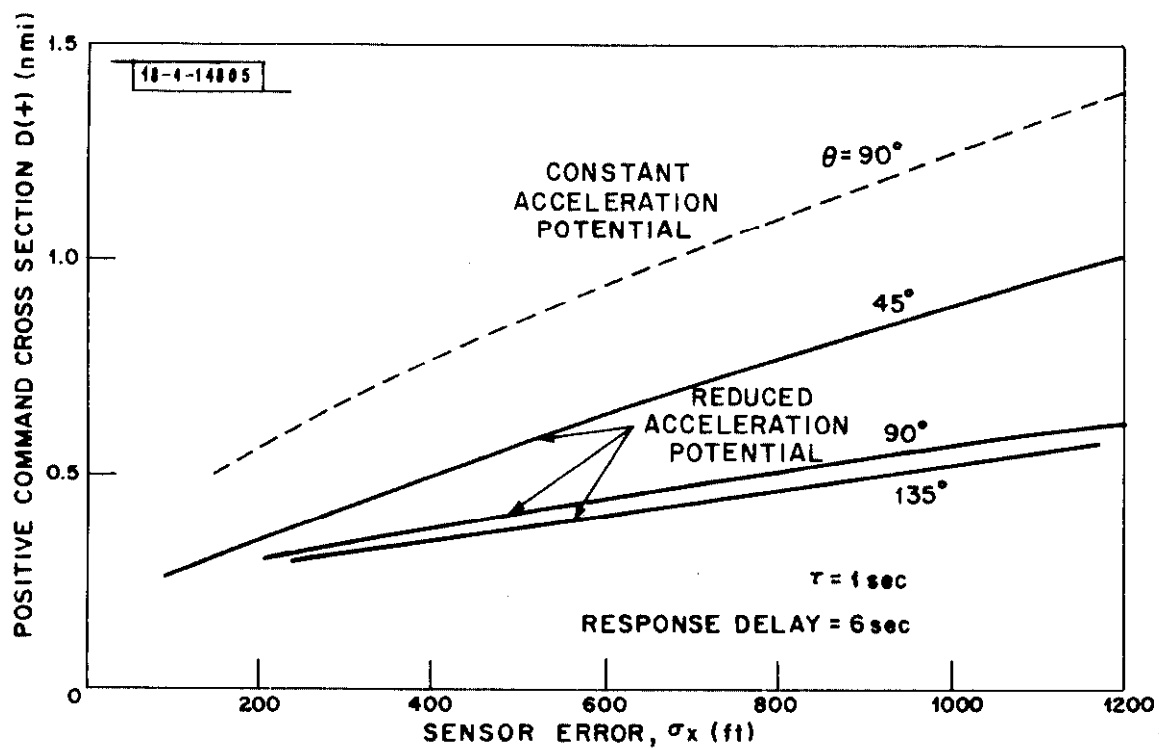


Figure 5-3. Command Cross Sections for Positive Commands.

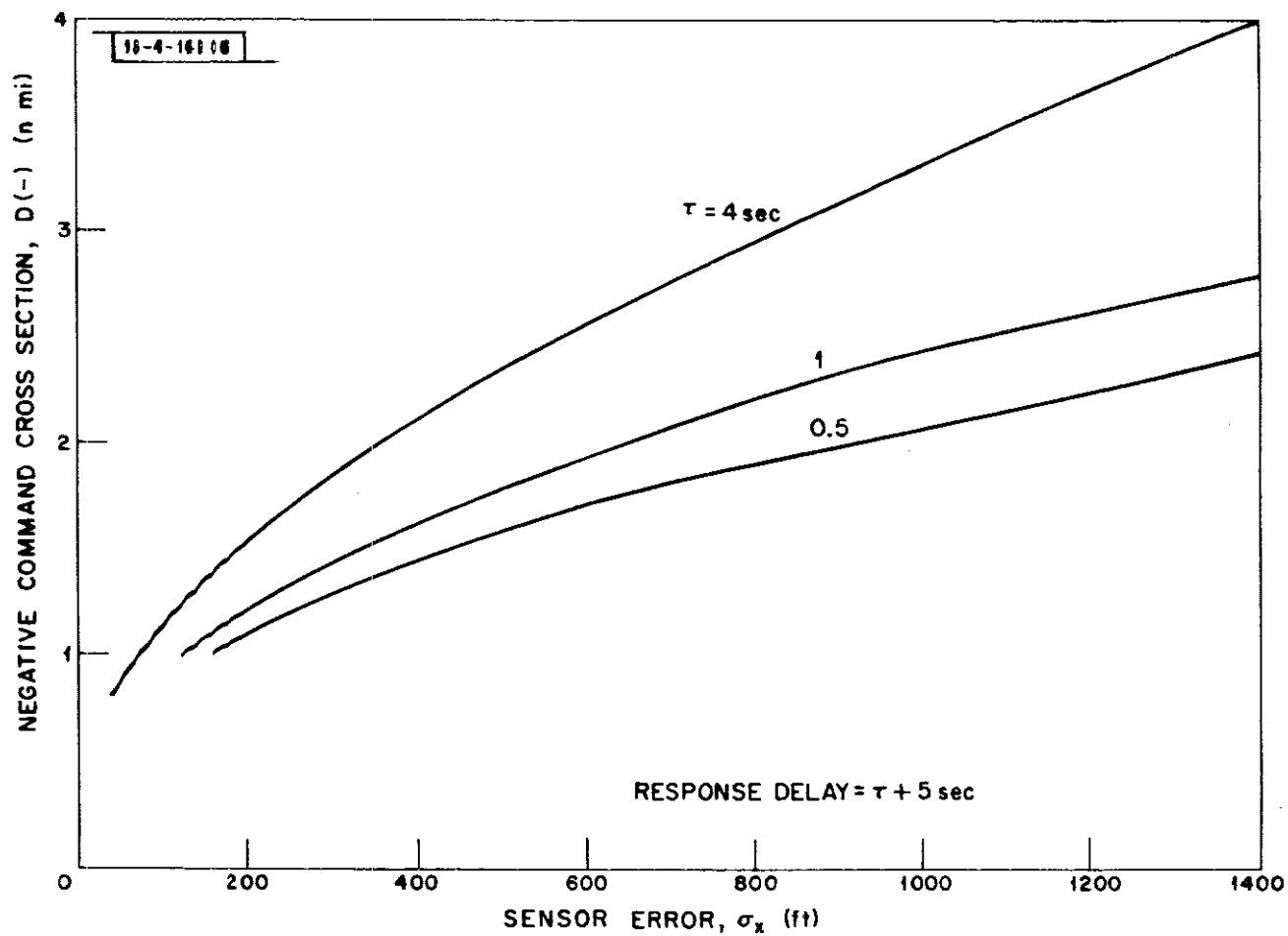


Figure 5-4. Effect of Update Interval on Negative Command Cross Section.

VI. CONCERNING THE DETERMINATION OF REQUIREMENTS

One reason for investigating the relationship between surveillance/communication system characteristics and IPC performance is to determine those features of the support systems to which IPC performance is most sensitive. This allows us to suggest improvements to the design of those systems which offer maximum performance benefits. But questions also arise concerning the level of improvement needed; for example, what level of tracking accuracy is required to make IPC feasible? Before attempting to determine such requirements, one would be well advised to scrutinize the available analytical techniques in light of the results of the previous sections.

Figure 6-1 illustrates the interaction of various factors in determining

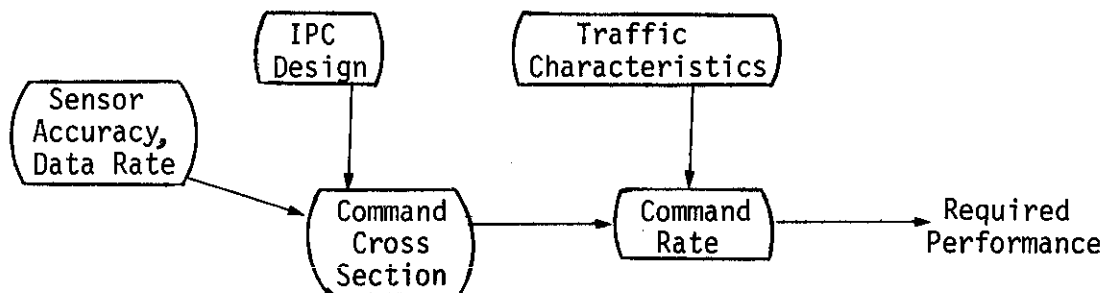


Figure 6-1. Factors in Determination of IPC Performance.

the IPC command rate, which we shall take for the moment as the most significant indicator of performance. The determination of surveillance requirements depends upon defining the required level of performance and working backward in the above sequence. We can reliably determine the required surveillance

quality only if uncertainties in the other factors are small, or if their impact on performance is negligible. Unfortunately, this is not the case. Three areas of significant uncertainty exist and are discussed below.

A. IPC Design

The IPC concept is still at an early stage of development. Many design options are open, and lack of previous experience with IPC-type systems results in very unsatisfactory knowledge concerning many design questions. Operational considerations, which can be properly assessed only with flight tests, can significantly affect IPC procedures and parameters. Different IPC configurations must be judged by different performance criteria, which complicates the setting of requirements.

B. Traffic Characteristics

In evaluating IPC performance, we are interested in the frequency of encounters and the geometries at which these encounters occur. The peak airborne count for various time periods can be predicted, but the peaking in density due to local peculiarities is difficult to anticipate. In addition, the significance of density variations is difficult to assess without careful investigation. A highly peaked density may not be significant for IPC purposes if the aircraft in the crowded area are normally under ground control or if some factor operates at that point to produce highly ordered traffic flow. Density peaks over small areas may be of minor significance if a single properly chosen positive command can guide aircraft through the crowded region. It is not sufficient merely to know the rate of commands experienced at a given point, since aircraft do not normally remain in a given area for extended periods of time. A more satisfactory approach to command rate would probably involve

determining the number of commands received in one terminal operation, or the probability that a command would interfere with some necessary flight maneuver. Use of such improved performance criteria would require that traffic patterns be specified in considerable detail and that the effect of IPC commands on flight progress be properly simulated. Unfortunately, the terminal area flight patterns giving rise to particular encounter geometries have never been clearly analyzed for current traffic, so little can be said of future patterns.

C. Required Performance

Finally, we do not know the level of performance which should be required. It is known that at some point, IPC commands will add an unacceptable burden to the pilot workload, produce excessive annoyance, and begin to interfere with flight progress. But, the acceptable region can be defined only vaguely. The uncertainties in IPC design have an effect here, in that the type of display employed and the required pilot responses are important in establishing the tolerable rate of commands.

As an example of the sensitivity of sensor requirements to other factors, consider the case where our intended operating point is on Curve B of Figure 5-1 at $\sigma_x = 600$ ft. This corresponds to a cross section $D(-) = 2.2$. Now, suppose that some factor (such as traffic density, traffic patterns, IPC design, etc.) produces an error of 25%, forcing the required $D(-)$ down to $D(-) = 2.2 \div 1.25 = 1.75$. The required value of σ_x is now $\sigma_x = 275$ ft, a decrease by more than a factor of 2! Thus, even minor errors in predicting the operational environment of the IPC system lead to major uncertainties in required sensor performance.

With the difficulties cited above in mind, we shall now work some examples which illustrate the range of results which can be expected in determining surveillance requirements. We shall use traffic predictions for the Los Angeles Basin (a 60 nmi x 120 nmi area) in the year 1995.³ The total two-dimensional density is $1365 \text{ aircraft}/7200 \text{ nmi}^2 = 0.190 \text{ aircraft/nmi}^2$. Assuming that only 1/5 of the aircraft are close enough in altitude to interact, we obtain an effective two-dimensional density $\rho = 0.190 \times 1/5 = 0.038 \text{ aircraft/nmi}^2$. Insertion of this value into the gas model equation (3-6) gives (for $V_0 = 180$ knots)

$$\lambda = 17.40 \quad .$$

If we now specify a required value for the average command rate, we can use this equation to find the required collision cross section. The curves of Figures 5-1 and 5-3 can then be used to obtain the corresponding sensor accuracy. Table 4 displays the results of this process for some representative combinations of command rate requirements and algorithmic parameters. If we assume for the moment that typical error variances of improved third generation sensors will be 300 - 400 ft, then it appears that sensors with update rates of one second or less are well suited for most of the situations considered. On the other hand, sensors with 4-second update intervals are inadequate in many cases.

Table 4. Accuracy Requirements - Selected Cases .

Required Average Command Rate	Required Cross Section	Curve Used for Cross Section	Required σ_x for $\tau=1$ second
$\lambda(-) = 90$	5.2 nmi	B, Figure 5-1	> 1200 ft
$\lambda(-) = 60$	3.4 nmi	B, Figure 5-1	660 ft
$\lambda(-) = 20$	1.15 nmi	B, Figure 5-1	unachievable
$\lambda(+) = 15$	0.86 nmi	$\theta=45^\circ$, Figure 5-3	940 ft
$\lambda(+) = 15$	0.86 nmi	$\theta=90^\circ$, Figure 5-3	>1200 ft
$\lambda(+) = 10$	0.58 nmi	$\theta=45^\circ$, Figure 5-3	530 ft
$\lambda(+) = 10$	0.58 nmi	$\theta=90^\circ$, Figure 5-3	1050 ft
$\lambda(+) = 5$	0.287 nmi	$\theta=45^\circ$, Figure 5-3	140 ft
$\lambda(+) = 5$	0.287 nmi	$\theta=90^\circ$, Figure 5-3	160 ft

VII. SUMMARY AND CONCLUSIONS

One of the most important indicators of IPC performance is the rate at which the pilot receives commands in a given traffic environment. We have seen that characteristics of the surveillance and communication systems can influence the command rate in several ways. The presence of tracking errors forces more conservative alarm criteria which generate commands more often. A higher command rate also results when warning times must be increased to accommodate communication delays. The rate of negative commands is especially sensitive to delays (Section V Part B) since the protected region grows rapidly as we extend the warning time.

For the IPC algorithm considered, it can also be said that the sensitivity to surveillance errors is moderate in the sense that substantial changes in sensor accuracy fail to drastically change the command rate. The converse statement is that if sensor accuracy is the parameter which is to be varied in order to meet a set of performance requirements, then minor revisions of those requirements will necessitate substantial changes in sensor accuracy.

In Section VI we discussed various difficulties involved in the evaluation of IPC performance. Certain features of IPC design and traffic environment must be clarified before we can accurately determine surveillance requirements. A major step forward would be the modeling of a realistic traffic environment which would reproduce details of actual flight patterns while allowing extension to various prediction epochs and geographical areas. Such a model would allow us to determine the frequency and geometry of encounters

in each region of the airspace for each type of user. Efforts should also be made to define the procedures and resolution strategies which may be used in the system design. Progress in this area may require cockpit simulations of IPC operation.

As far as various features of the Discrete Address Beacon System (DABS) are concerned, the one design choice which is certain to make a substantial impact upon IPC is that between phased array and rotating antenna interrogators. The increased data rate achievable with the phased array improves tracking accuracy. At the same time, the required warning time is reduced by faster message delivery.

A conservative view of the results of Section VI would lead to the conclusion that by 1995, a phased array interrogator would be required for IPC in some high density areas. But, until the details of IPC design and operation are clarified, there remains the possibility that this is an overly-conservative appraisal. If a phased array is not available, there remains the possibility of altering the IPC design and traffic environment in order to allow use of a high performance rotating antenna interrogator.

In closing, we shall call attention once more to the many simplifications and assumptions employed in generating the quantitative data in this report. Among the improvements in technique which could be made with additional effort are:

- (1) a stochastic simulation of error inputs (rather than semi-deterministic calculations)
- (2) more realistic modeling of sensor error and tracking geometry
- (3) use of improved tracking algorithms (including "adaptive" trackers)
- (4) improved resolution logic
- (5) a better determination of safety level.

APPENDIX A: TRACKING MODEL

The tracking algorithm used to model tracking errors employs a simple curve-fitting technique which provides the least squared error for rectilinear motion. The equations are formulated in one dimensional form and applied separately in each coordinate. Aircraft acceleration can introduce certain bias errors which are explained in Subsections B and C below. In Subsection D the method of determining prediction errors is discussed.

A. Rectilinear Motion

Suppose that we desire to measure the current position u and current velocity \dot{u} in a given coordinate using the last N observations of the aircraft position. For rectilinear motion, the n^{th} observation is of the form

$$y_n^* = u - \dot{u}\tau (N - n) + \epsilon_n \quad (\text{A-1})$$

where τ is the interval between observations (assumed constant) and ϵ_n is the measurement error for the n^{th} observation. We assume that $E[\epsilon_n] = 0$ and

$$E[\epsilon_m \epsilon_n] = \begin{cases} \sigma_x^2 & \text{if } m=n \\ 0 & m \neq n \end{cases}$$

The least-square error estimators can readily be shown to be

$$\hat{u} = \frac{2}{N} \sum_{n=1}^N \left(3 \frac{n}{N+1} - 1 \right) y_n^* \quad (\text{A-2})$$

$$\dot{u} = \frac{6}{N(N-1)\tau} \sum_{n=1}^N \left(\frac{2n}{N+1} - 1 \right) y_n^* \quad (A-3)$$

The necessary variances and covariances are calculated by inserting (A-1) into (A-2) and (A-3). They are

$$\begin{aligned} \sigma_u^2 &= \frac{2 \sigma_x^2 \tau}{N} \cdot \frac{2N-1}{N+1} \cong \frac{4 \sigma_x^2 \tau}{1} \\ \sigma_{\dot{u}}^2 &= \frac{12 \sigma_x^2}{N \tau^2} \cdot \frac{1}{N^2-1} \cong \frac{12 \sigma_x^2 \tau}{\tau^3} \\ \sigma_{u\dot{u}}^2 &= \frac{6 \sigma_x^2}{N \tau} \cdot \frac{1}{N+1} \cong \frac{6 \sigma_x^2 \tau}{\tau^2} \end{aligned} \quad (A-4)$$

where $T = (N-1)\tau$ is the time interval between the first and last observations.

B. Accelerated Motion

Suppose now that we employ the same estimators, but that the aircraft motion is not rectilinear. The worst-case deviation can be ascribed to a constant acceleration U in the coordinate under consideration. Observations are now of the form

$$Y_n^* = u - \dot{u}\tau(N-n) + \frac{U}{2} \tau^2 (N-n)^2 \quad (A-5)$$

The output of our estimator is now

$$\hat{u} = u - \frac{U \tau^2}{12} (N - 2) (N - 1) + \frac{2}{N} \sum_{n=1}^N \left(3 \frac{n}{N+1} - 1\right) \epsilon_n \quad (\text{A-6})$$

$$\dot{\hat{u}} = \dot{u} - \frac{U \tau}{2} (N - 1) + \frac{6}{N(N-1)\tau} \sum_{n=1}^N \left(\frac{2n}{N+1} - 1\right) \epsilon_n \quad (\text{A-7})$$

We see that in addition to a noise error output there is a bias output which is proportional to U . The magnitudes of the biases can be written (for $U > 0$) as

$$B_u = \frac{U \tau^2}{12} (N - 1) (N - 2) = \frac{U T^2}{12} \quad (\text{A-8})$$

$$B_{\dot{u}} = \frac{U \tau}{2} (N - 1) = \frac{U T}{2} \quad (\text{A-9})$$

where T is the observation time. Note that while the noise error term decreases with the number of observations used, the bias error increases. We must therefore use some discretion in deciding how many observations we should employ in our estimation procedure. Suppose we define a bounding error which consists of the worst-case bias plus three times the standard deviation due to noise. The velocity error is then approximately

$$\dot{u}_\epsilon = \frac{U T}{2} + 3 \sqrt{\frac{12 \sigma_x^2 \tau}{T^3}} \quad (A-10)$$

The value of the observation time which minimizes this error is

$$T = \frac{3}{U} (4 \tau \sigma_x^2 U^3)^{1/5} \quad (A-11)$$

The bounding errors ("three sigma") for this value of T are

$$u_\epsilon = 4.33 (\tau^2 \sigma_x^4 U)^{1/5} \quad (A-12)$$

$$\dot{u}_\epsilon = 3.29 (\tau \sigma_x^2 U^3)^{1/5} \quad (A-13)$$

C. Change in Acceleration Potential

We shall now consider the case in which some event leads to a change in the acceleration potential of the aircraft. Suppose that N data points are employed with a potential acceleration U_1 for points 1 through M and a potential acceleration U_2 for data points M to N. The equations of motion now become

$$Y_n = u - u\tau N + \frac{U_1\tau^2 M^2}{2} + \frac{U_2\tau^2 (N^2 - M^2)}{2} + n \left[\dot{u}\tau - U_1\tau^2 M - U_2\tau^2 (N-M) \right] \\ + n^2 \left[\frac{U_1\tau^2}{2} \right] \quad 0 \leq n \leq M$$

$$Y_n = u - \dot{u}\tau N + \frac{U_2\tau^2 N^2}{2} + n \left[\dot{u}\tau - U_2\tau^2 N \right] + n^2 \left[\frac{U_2\tau^2}{2} \right] \quad M \leq n \leq N$$

The biases are now

$$B_u = \frac{U_1\tau^2}{12} \left[\frac{2M(2M-1)(M-1)(N+1) - 3M^2(M^2-1)}{N(N+1)} \right] \\ + \frac{U_2\tau^2}{12} \left[(N-1)(N-2) + \frac{M(M-1)}{N(N+1)} (3M(M+1) - 2(2M-1)(N+1)) \right]$$

and

$$B_{\dot{u}} = \frac{U_1\tau}{2} \left[\frac{M(N+1)(2M-1)(M-1) - M^2(M^2-1)}{N(N^2-1)} \right] \\ + \frac{U_2\tau}{2} \left[\frac{(2N^3 - 3N^2 + N - 2M^3 + 3M^2 - M)(N+1) - N^4 + M^4 + N^2 - M^2}{N(N^2-1)} \right]$$

D. Extrapolation of Tracking Errors

The error in estimating the future position of the aircraft arises from two terms, one due to the acceleration bias and the other due to noise in position measurements. The bias error will be projected according to the equation

$$\epsilon_{\text{bias}} = B_u + B_{\dot{u}} T_f$$

where T_f is the time into the future through which we extrapolate the track.

The position error due to the noise term is expressed in terms of its variance. For a rectilinear projection

$$\sigma^2(T_f) = E [(\hat{u} + \hat{\dot{u}} T_f - (u + \dot{u} T_f))^2] = \sigma_u^2 + 2T_f \sigma_{u\dot{u}}^2 + T_f^2 \sigma_{\dot{u}}^2 .$$

This expression is also a good approximation for projections along curved paths. The noise term is then set equal to $3\sigma(T_f)$ and the total projected uncertainty becomes

$$\epsilon_{\text{total}} = \epsilon_{\text{bias}} + \epsilon_{\text{noise}} = B_u + B_{\dot{u}} T_f + 3 \sigma(T_f)$$

The extreme simplicity of this formulation is due to the assumptions of isotropy in measurement errors and acceleration potential.

In determining the variance in the aircraft separation, we note that the separation may be written $\vec{D} = \vec{p}_2 - \vec{p}_1$ where \vec{p}_i is the position of aircraft i . If the errors in \vec{p}_1 and \vec{p}_2 are of zero mean and are statistically independent, then $\text{Cov}(\vec{D}) = \text{Cov}(\vec{p}_2) + \text{Cov}(\vec{p}_1)$. Thus, for a given coordinate, the variance in separation is the sum of the variances in the positions of the individual aircraft.

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2. McIvor, D.E. and Matney, J.S. "Analysis of Intermittent Positive Control Conflict Rates and Computer Processing," MITRE Corporation, Technical Report MTR-6175, 7 April 1972.
3. See Reference 1, Vol. 1, p. 68.