Program Generation with Spiral:
Beyond Transforms

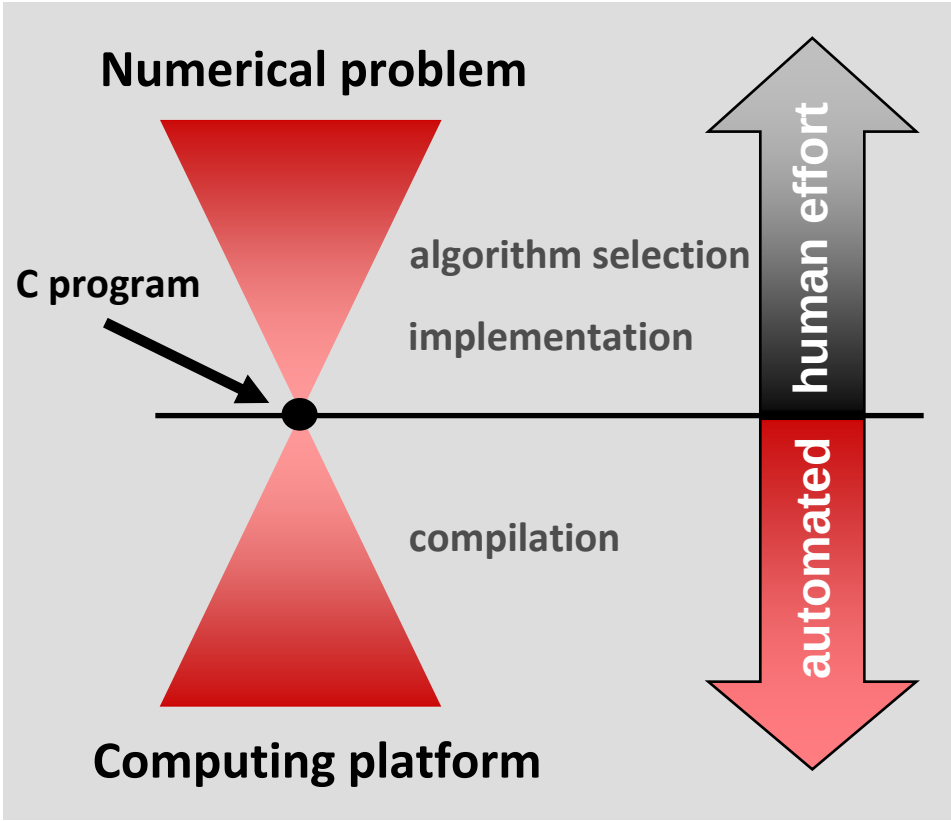
Franz Franchetti, Daniel Mcfarlin, Frédéric de Mesmay, Hao Shen, Tomasz W. Włodarczyk, Srinivas Chellappa, Marek R. Telgarsky, Peter A. Milder, Yevgen Voronenko, Qian Yu, James C. Hoe, José M. F. Moura, Markus Püschel

Electrical and Computer Engineering
Carnegie Mellon University

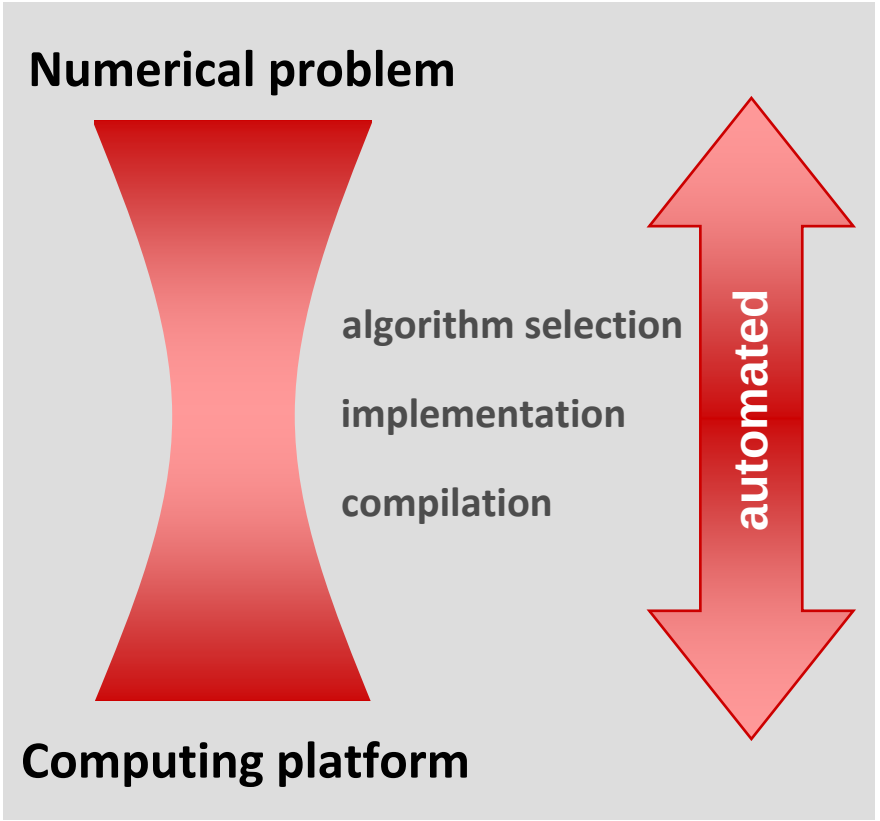
This work was supported by
DARPA DESA program, NSF-NGS/ITR, NSF-ACR, Mercury Inc., and Intel

Vision Behind Spiral

Current



Future

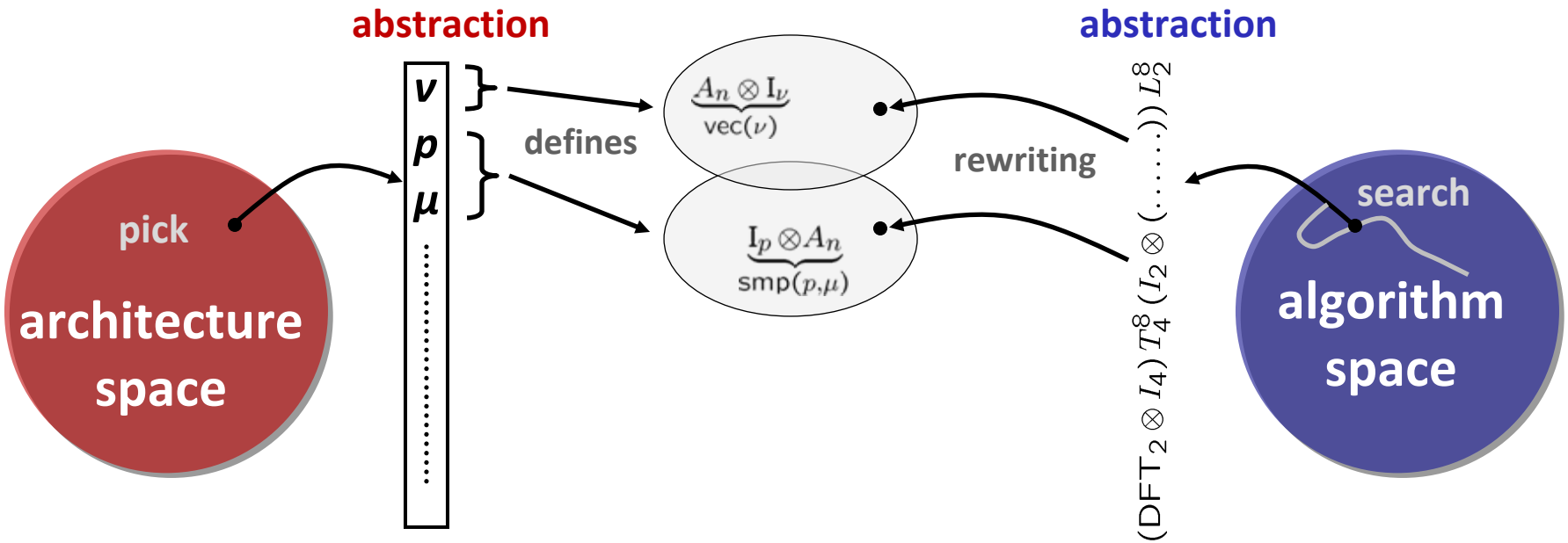


C code a singularity: Compiler has no access to high level information

Challenge: conquer the high abstraction level for complete automation

Main Idea: Program Generation

Model: common abstraction
= spaces of matching formulas



Architectural parameter:
Vector length,
#processors, …

optimization

Kernel:
problem size,
algorithm choice

Expressing Kernels as Operator Formulas

Matrix-Matrix Multiplication

$$\mathrm{MMM}_{1,1,1} \rightarrow (\cdot)_1$$

$$\mathrm{MMM}_{m,n,k} \rightarrow (\otimes)_{m/m_b \times 1} \otimes \mathrm{MMM}_{m_b,n,k}$$

$$\mathrm{MMM}_{m,n,k} \rightarrow \mathrm{MMM}_{m,nb,k} \otimes (\otimes)_{1 \times n/nb}$$

$$\mathrm{MMM}_{m,n,k} \rightarrow ((\Sigma_{k/k_b} \circ (\cdot)_{k/k_b}) \otimes \mathrm{MMM}_{m,n,k_b}) \circ ((L^{mk/k_b}_{k/k_b} \otimes I_{k_b}) \times I_{kn})$$

$$\mathrm{MMM}_{m,n,k} \rightarrow (L^{mn/n_b}_{m} \otimes I_{n_b}) \circ ((\otimes)_{1 \times n/n_b} \otimes \mathrm{MMM}_{m,n_b,k}) \circ (I_{km} \times (L^{kn/n_b}_{n/n_b} \otimes I_{n_b}))$$

Viterbi Decoder

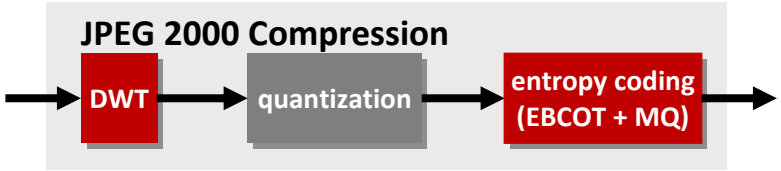
$$\underbrace{\mathrm{Vit}}_{\mathrm{vec}(v)} \rightarrow \underbrace{\left(\prod (L \times I) \circ (I \otimes C)\right) \circ Id}_{\mathrm{vec}(v)}$$

$$\rightarrow \left(\prod \underbrace{(L \times I) \circ (I \otimes C)}_{\mathrm{vec}(v)}\right) \circ Id$$

$$£ \rightarrow \left(\prod (L \otimes I_v \times I) \circ (I \otimes C \otimes I_v) \circ (\vec{L} \times I)\right) \circ Id$$

$$\rightarrow \prod (L \otimes I_v \times I) \circ (I \otimes (B \otimes I_v)) \circ (\vec{L} \times I)$$

JPEG 2000 (Wavelet, EBCOT)



$$\mathrm{SC}(\chi_{m,n}, \sigma_{m,n}) : (\mathbb{Z}_2^9 \times \mathbb{Z}_2^9) \rightarrow (\mathbb{N}, \mathbb{Z}_2)$$

$$(I \times \mathrm{xor}_2) \circ (T_{SC} \times I) \circ (H \times V \times I) \circ (\underline{L^2_4} \times G_4) \circ (\underline{\begin{pmatrix} 1 \\ 1 \end{pmatrix}} \times \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix})$$

$$\mathrm{H, V} : (\mathbb{Z}_2^9 \times \mathbb{Z}_2^9) \rightarrow \mathbb{N}$$

$$\mathrm{H:} \quad h \circ (f \times f) \circ (G_1 \times \mathrm{C}_{-2} \times G_1 \times G_7 \times \mathrm{C}_{-2} \times G_7) \circ \underline{L^2_4} \circ (\underline{\binom{1}{1}} \times \underline{\binom{1}{1}})$$

$$\mathrm{V:} \quad h \circ (f \times f) \circ (G_3 \times \mathrm{C}_{-2} \times G_3 \times G_5 \times \mathrm{C}_{-2} \times G_5) \circ \underline{L^2_4} \circ (\underline{\binom{1}{1}} \times \underline{\binom{1}{1}})$$

$$f : \mathrm{mul}_2 \circ (I \times \mathrm{sub}_2) \circ (I \times \mathrm{C}_1 \times \mathrm{mul}_2)$$

$$h : \mathrm{min}_2 \circ (\mathrm{C}_1 \times \mathrm{max}_2) \circ (\mathrm{C}_{-1} \times \mathrm{sum}_2)$$

Synthetic Aperture Radar (SAR)

$$\begin{aligned}
\mathrm{SAR} &\rightarrow \mathrm{2D\text{-}iDFT} \circ \mathrm{Interpl} \circ \mathrm{MatchFilt} \circ \mathrm{prep} \\
\mathrm{2D\text{-}iDFT} &\rightarrow \mathrm{iDFT} \otimes \mathrm{iDFT} \\
\mathrm{MatchFilt} &\rightarrow \mathrm{Filt} \circ (\mathrm{I} \times \mathrm{C}_f) \\
\mathrm{Filt} &\rightarrow (\mathrm{I} \otimes (\cdot)) \\
\mathrm{Interpl} &\rightarrow (\Sigma \otimes \mathrm{I}) \circ (\mathrm{I} \otimes_j \mathrm{S}_{x_j \otimes y_j}) \circ \mathrm{Filt} \circ ((\mathrm{I} \otimes \mathbf{1} \otimes \mathrm{I}) \times \mathrm{I}) \circ (\mathrm{I} \times \mathrm{C}_{\imath \otimes_j g_j})
\end{aligned}$$