Resource-aware Distributed Block-based LU Decomposition on Wireless Sensor Networks

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Block-based LU

Any *N* x *N* matrix A can be partitioned into 4 matrices

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{bmatrix}$$

where A_{11} is b x b matrix
A_{12} is b x (N-b) matrix
A_{21} is (N-b) x b matrix
A_{22} is (N-b) x (N-b) matrix

Perform:

- **LU decomposition (Crout's algorithm) over A_{11}**

$$A_{11} = L_{11} \cdot U_{11}$$

- **Upper decomposition over A_{12}**

$$A_{12} = L_{11} \cdot U_{12}$$

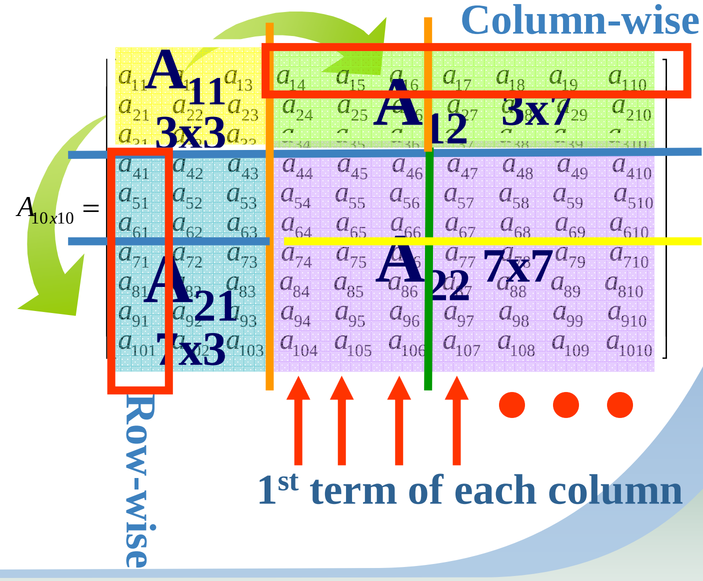
- **Lower decomposition over A_{21}**

$$A_{21} = L_{21} \cdot U_{11}$$

- **Repeat the partitioning for A' or apply Crout's algorithm**

$$A' = A_{22} - L_{21} \cdot U_{12}$$

For 10x10 Matrix A, and b = 3



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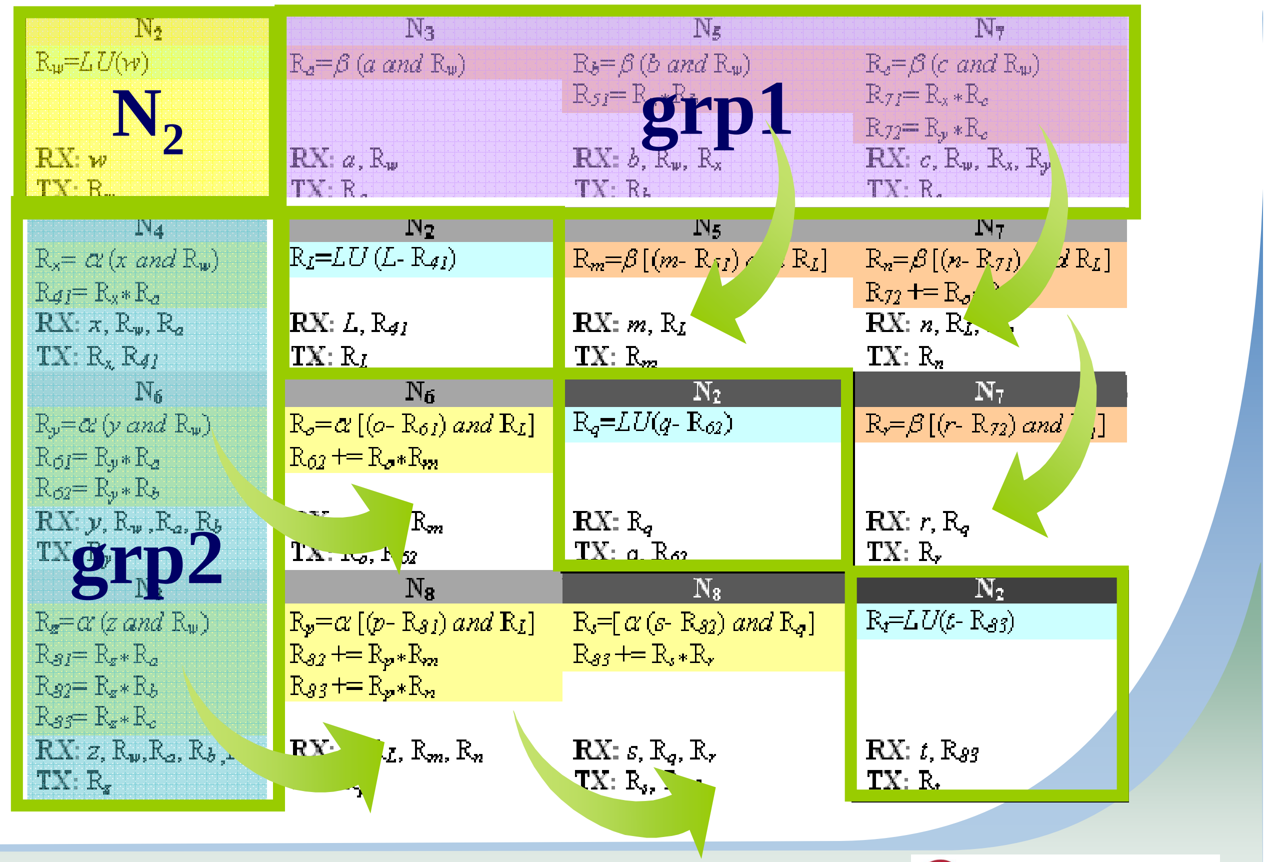
<u>Node N2</u>
- **responsible for LU matrix operation**

<u>Group1</u>:
- **Nodes responsible for upper matrix computation (β operation)**
- **Nodes have odd addresses**

<u>Group2</u>
- **Nodes responsible for lower matrix computation (α operation)**
- **Nodes have even addresses**

N = 13, b = 3

#nodes = 2. [N/b] -1 = 7



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Round 3

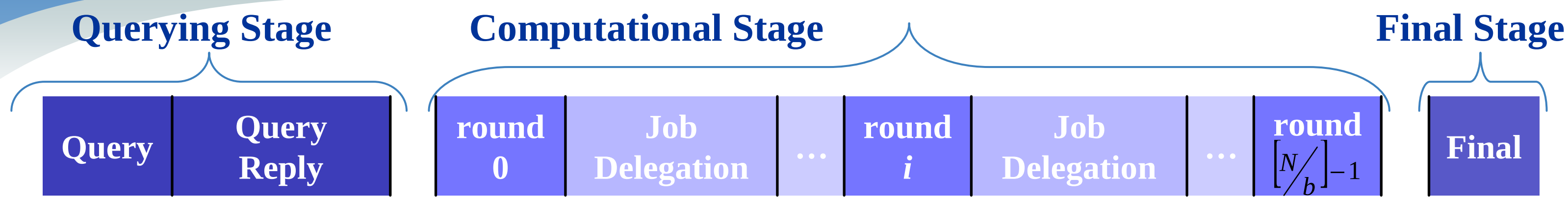
w_{3x3}	a_{3x3}	b_{3x3}	c_{3x4}
x_{3x3}	l_{3x3}	m_{3x3}	n_{3x4}
y_{3x3}	o_{3x3}	q_{3x3}	r_{3x4}
z_{4x3}	p_{4x3}	s_{4x3}	t_{4x4}

N2 $R_w=LU(w)$ RX: w TX: R_w	**N3** $R_a=\beta\ (a \text{ and } R_w)$ RX: a, R_w TX: R_a	**N5** $R_b=\beta\ (b \text{ and } R_w)$ $R_{51}=R_x*R_b$ RX: b, R_w, R_x TX: R_b	**N7** $R_c=\beta\ (c \text{ and } R_w)$ $R_{71}=R_x*R_c$ $R_{72}=R_y*R_c$ RX: c, R_w, R_x, R_y TX: R_c
N4 $R_x=\alpha\ (x \text{ and } R_w)$ $R_{41}=R_x*R_a$ RX: x, R_w, R_a TX: R_x, R_{41}	**N2** $R_l=LU\ (l- R_{41})$ RX: l, R_{41} TX: R_l	**N5** $R_m=\beta\ [(m- R_{51}) \text{ and } R_l]$ RX: m, R_l TX: R_m	**N7** $R_n=\beta\ [(n- R_{71}) \text{ and } R_l]$ R_{72} += R_o*R_n RX: n, R_l, R_o TX: R_n
N6 $R_y=\alpha\ (y \text{ and } R_w)$ $R_{61}=R_y*R_a$ $R_{62}=R_y*R_b$ RX: y, R_w, R_a, R_b TX: R_y	**N6** $R_o=\alpha\ [(o- R_{61}) \text{ and } R_l]$ R_{62} += R_o*R_m RX: o, R_l, R_m TX: R_o, R_{62}	**N2** $R_q=LU(q- R_{52})$ RX: R_q TX: q, R_{62}	**N7** $R_r=\beta\ [(r- R_{72}) \text{ and } R_q]$ RX: r, R_q TX: R_r
N8 $R_z=\alpha\ (z \text{ and } R_w)$ $R_{81}=R_z*R_a$ $R_{82}=R_z*R_b$ $R_{83}=R_z*R_c$ RX: z, R_w, R_a, R_b, R_c TX: R_z	**N8** $R_p=\alpha\ [(p- R_{81}) \text{ and } R_l]$ R_{82} += R_p*R_m R_{83} += R_p*R_n RX: p, R_l, R_m, R_n TX: R_p	**N8** $R_s=[\alpha\ (s- R_{82}) \text{ and } R_q]$ R_{83} += R_s*R_r RX: s, R_q, R_r TX: R_s, R_{83}	**N2** $R_t=LU(t- R_{83})$ RX: t, R_{83} TX: R_t

#Rounds = [N/b]
= 4

Round *i*

- BS **transmits** to grp1, grp2 and N2
- Each node in grp1 / grp2 **broadcasts** its result
- The 1st node in grp2 **unicasts** 1 message to N2
- At the end of each round, the 1st nodes in each group can **RETIRE**



1.Querying stage:

- **BS broadcasts QUERY packet to query the voltage level and the amount of available memory**
- **BS selects 2*[N/b]-1 nodes to be active and the rest to standby (used in a failure management scheme through auto-power notification)**

2.Computation Stage: It is formed of [N/b] rounds and [N/b] – 1 job delegation periods

3.Final Stage: BS regroups the result

Each node infers its time slot from its ID

➢ **Energy and Storage-aware Job Allocation:**

Divide voltage levels 3 levels

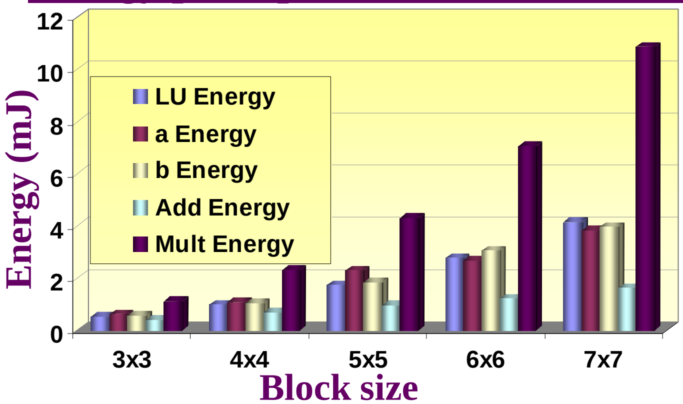
- **10% < $V1 = V_{min}$ < 30% of the battery voltage**
- **$V_{min} < V2 < V_{max}$ of the battery voltage**
- **$V3 = V_{max}$ > 70% of the battery voltage**

Divide amount of free memory into 3 levels

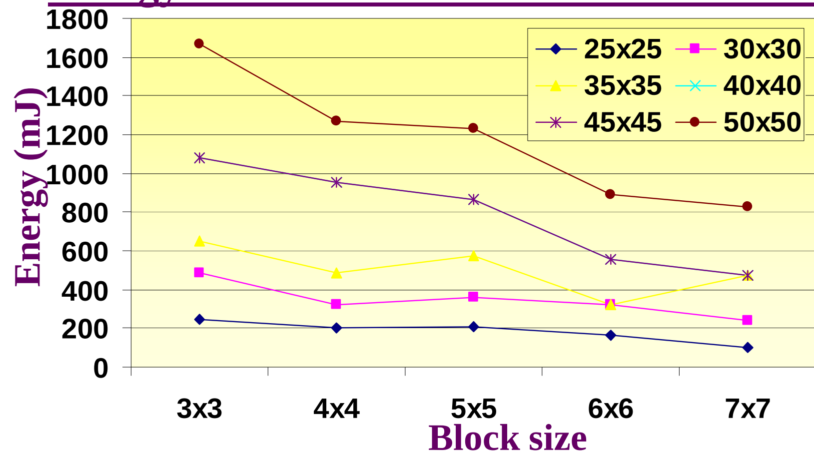
- **10% < $M1 = M_{min}$ < 30% of memory**
- **$M_{min} < M2 < M_{max}$ of memory**
- **$M3 = M_{max}$ > 70% of memory**

ENERGY and STORAGE requirements increase with the iteration number

Energy per Operation vs. Matrix Size



Energy for diff. matrix sizes and block sizes



$P_{avg} \approx 4.06 \times 10^{-2}$W

E_{max} = 10.87 mJ, E_{min}=0.51 mJ

P_{max} ranged between 4.09×10^{-2}W and 4.12×10^{-2}W

Contributions

- ✓ **Proposed a scheme for distributing block-based LU decomposition on WSN**
- ✓ **Proposed an energy and resource-aware job allocation**
- ✓ **Proposed a mostly-decentralized synchronous MAC / Time Slot Allocation**
- ✓ **Proposed a failure management scheme through auto-power notification**
- ✓ **Proved that LU decomposition requirements are feasible on WSN**