Resource-aware Distributed Block-based LU Decomposition on Wireless Sensor Networks

By Sherine Abdelhak, Soumik Ghosh, Jared Tessier, Magdy Bayoumi

CACS, University of Louisiana at Lafayette
Lafayette, LA 70504
{spa9242, sxg5317, jst2777, mab} @ cacs.louisiana.edu

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Any $N \times N$ matrix $A$ can be partitioned into 4 matrices:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{bmatrix}$$

where $A_{11}$ is $b \times b$ matrix

$A_{12}$ is $b \times (N-b)$ matrix

$A_{21}$ is $(N-b) \times b$ matrix

$A_{22}$ is $(N-b) \times (N-b)$ matrix

Perform:

- LU decomposition (Crout’s algorithm) over $A_{11}$

  $$A_{11} = L_{11} \cdot U_{11}$$

- Upper decomposition over $A_{12}$

  $$A_{12} = L_{11} \cdot U_{12}$$

- Lower decomposition over $A_{21}$

  $$A_{21} = L_{21} \cdot U_{11}$$

- Repeat the partitioning for $A'$ or apply Crout’s algorithm

  $$A' = A_{22} - L_{21} \cdot U_{12}$$

For 10x10 Matrix $A$, and $b = 3$
**Resource-aware Distributed Block-based LU Decomposition on Wireless Sensor Networks**

**Node N2**
- responsible for LU matrix operation

**Group1:**
- Nodes responsible for upper matrix computation (β operation)
- Nodes have **odd addresses**

<table>
<thead>
<tr>
<th>R_{2}</th>
<th>L_{2} + R_{2}</th>
<th>R_{2}</th>
<th>L_{2}</th>
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<th>L_{2} + R_{2}</th>
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<tbody>
<tr>
<td>N_{2}</td>
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<td>\text{TX: } R_{2}</td>
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</table>

**Group2**
- Nodes responsible for lower matrix computation (α operation)
- Nodes have **even addresses**

<table>
<thead>
<tr>
<th>R_{1}</th>
<th>L_{1} + R_{1}</th>
<th>R_{1}</th>
<th>L_{1}</th>
<th>R_{1}</th>
<th>L_{1} + R_{1}</th>
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\#nodes = 2. \left[ \frac{N}{b} \right] - 1 = 7

\( N = 13, b = 3 \)
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**Introduction**

- BS transmits to grp1, grp2 and N2
- Each node in grp1 / grp2 broadcasts its result
- The 1st node in grp2 unicasts 1 message to N2
- At the end of each round, the 1st nodes in each group can RETIRE

**Proposed Scheme**

- #Rounds = \([N/b]\) = 4
- BS transmits to grp1, grp2 and N2
- Each node in grp1 / grp2 broadcasts its result
- The 1st node in grp2 unicasts 1 message to N2
- At the end of each round, the 1st nodes in each group can RETIRE
1. Querying stage:
   - BS broadcasts QUERY packet to query the voltage level and the amount of available memory
   - BS selects 2*[N/b]-1 nodes to be active and the rest to standby (used in a failure management scheme through auto-power notification)

2. Computation Stage: It is formed of [N/b] rounds and [N/b] – 1 job delegation periods

3. Final Stage: BS regroups the result

Each node infers its time slot from its ID

Energy and Storage-aware Job Allocation:

- Divide voltage levels 3 levels
  - 10% < V1 = V_{min} < 30% of the battery voltage
  - V_{min} < V2 < V_{max} of the battery voltage
  - V3 = V_{max} > 70% of the battery voltage

- Divide amount of free memory into 3 levels
  - 10% < M1 = M_{min} < 30% of memory
  - M_{min} < M2 < M_{max} of memory
  - M3 = M_{max} > 70% of memory

ENERGY and STORAGE requirements increase with the iteration number
Energy per Operation vs. Matrix Size

\[
P_{\text{avg}} \approx 4.06 \times 10^{-2} \text{W}
\]
\[
P_{\text{max}} \text{ ranged between } 4.09 \times 10^{-2} \text{W and } 4.12 \times 10^{-2} \text{W}
\]

\[
E_{\text{max}} = 10.87 \text{ mJ}, E_{\text{min}} = 0.51 \text{ mJ}
\]

Contributions

✓ Proposed a scheme for distributing block-based LU decomposition on WSN
✓ Proposed an energy and resource-aware job allocation
✓ Proposed a mostly-decentralized synchronous MAC / Time Slot Allocation
✓ Proposed a failure management scheme through auto-power notification
✓ Proved that LU decomposition requirements are feasible on WSN