Efficient Multidimensional Polynomial Filtering for Nonlinear Digital Predistortion

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Introduction

The Volterra kernel is a multidimensional, polynomial extension to the linear filter that can be used to model arbitrary nonlinear systems. While theoretically significant, its complexity grows exponentially with polynomial order [1]. For input $x(n)$, its output is given by

$$y_V(n) = \sum_{p=1}^{P} \sum_{k_1=0}^{M-1} \cdots \sum_{k_p=0}^{M-1} h_p(k_1, \cdots, k_p) \prod_{\ell=1}^{p} x(n - k_\ell).$$

Due to its high computational complexity, most practical polynomial filters consider only subsets of the kernel. A memory polynomial [2], for example, uses only the Volterra kernel coefficients along the main diagonal, i.e.,

$$y_{MP}(n) = \sum_{p=1}^{P} \sum_{k=0}^{M-1} h_p(k, \cdots, k) x^p(n - k).$$

Other architectures such as the diagonal coordinate system (DCS) [3] and horizontal coordinate system (HCS) [4] use one-dimensional subkernels as building blocks for a larger nonlinear system which are chosen to achieve high performance and low computational complexity.

In all of these cases, one-dimensional coefficient swaths of the kernel are used, taking advantage of the ability to perform fast one-dimensional convolutions. Using multidimensional filters, however, has advantages. Filtering in multiple dimensions more easily addresses asymmetric nonlinearities that sometimes occur in practice. To generalize the division of the Volterra kernel to use multidimensional subkernels, we have created the cube coefficient subspace (CCS) architecture [5], which builds a polynomial signal processor with small nonlinear filters of arbitrary dimension and polynomial order. This allows us to adapt our system in more dimensions with fewer components, which leads to lower computational complexity for a given modeling error performance criterion.

In this abstract, we present the CCS architecture and use it to create a nonlinear digital predistorter (NDP) for reducing adjacent channel interference (ACI) caused by a solid state power amplifier (PA) in a Q-band satellite communication (SATCOM) system. For narrowband applications, a suffi-

cient NDP can be implemented with a computationally efficient look-up table (LUT), but as bandwidths increase, the memory effects imparted by the PA increase, and LUT performance degrades. On our amplifier of interest, our CCS NDP achieves greater ACI reduction than an LUT with lower computational complexity than other pruned Volterra kernel architectures.

Cube Coefficient Subspaces

The CCS architecture allows the user to partition the computationally complex, overparameterized Volterra kernel into smaller subkernels to increase computational efficiency without sacrificing performance. The components of the architecture are small hypercubes of arbitrary dimension within the full Volterra kernel coefficient space. The fact that the subkernels are multidimensional allows us to mitigate certain kinds of nonlinearities more efficiently than one-dimensional subkernels like HCS and DCS.

A pth-order CCS component performs a small d-dimensional convolution on d-fold products of the input signal. A CCS component has associated with it a polynomial order p, a dimension d, a memory depth M, and delays α_ℓ, for $1 \le \ell \le p$. The output of a CCS component is given by

$$y_{CCS}(n) = \sum_{k_1=0}^{M-1} \cdots \sum_{k_d=0}^{M-1} h(k_1, \cdots, k_d) \prod_{\ell=1}^{d} x(n - k_\ell - \alpha_\ell) \prod_{\hat{\ell}=d+1}^{p} x(n - \alpha_{\hat{\ell}}).$$

In Figure 1 we show the hardware layout of a 2-dimensional 5th-order CCS component with memory 3 in each dimension.
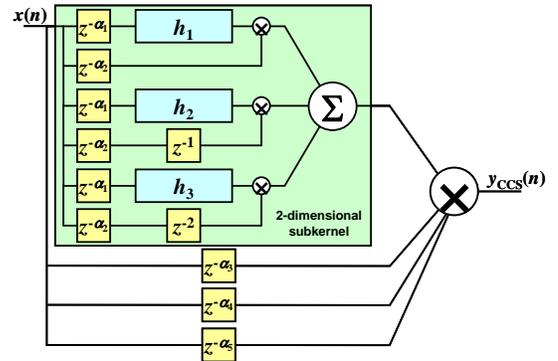
Figure 1: Hardware View of a CCS Component

We have also created a diagonal analog to CCS, the diagonal cube coefficient subspace (CCS-D), where each compo-

nent is a parallelepiped of arbitrary dimension in the full coefficient space. This allows us to efficiently create a nonlinear system that resides mostly along diagonal swaths of the Volterra coefficient space, but also maintain the benefits of multidimensional filtering. The output of a CCS-D with component memory M, dimension d and order p is given by

$$y_{\text{CCS-D}}(n) = \sum_{k_1=0}^{M-1} \cdots \sum_{k_d=0}^{M-1} h(k_1, \cdots, k_d) x(n-k_1-\alpha_1) \prod_{\ell=2}^{d} x(n-k_1-k_\ell-\alpha_\ell) \prod_{\ell=d+1}^{p} x(n-k_1-\alpha_\ell).$$

A multidimensional CCS component is less computationally efficient per coefficient than a 1-dimensional subkernel. A CCS component with memory M, dimension d and order p will have M^d coefficients and will require $p - d + \sum_{\ell=1}^{d} M^\ell$ multiplications and M^d additions. An HCS or DCS component with M^d coefficients also requires M^d additions, but requires only $M^d + p - 1$ multiplications. Indeed, as we move into higher dimensions, our complexity is exponential with respect to the dimension of our subkernel. For the nonlinearities we are interested in attacking, however, we can still achieve a less computationally complex NDP because we can use fewer components to maintain the same level of performance.

Simulation and Results

We use the inverse Volterra method [6] in conjunction with the greedy search method described in [7] to identify a CCS configuration for our NDP. In the greedy selection process the multidimensional nature of CCS components is advantageous; we can adapt in multiple dimensions from the first component selected.

In this abstract, we are applying our NDP at baseband, and as such we use the complex baseband version of the Volterra kernel [8], i.e., letting $O(P) = \{1, 3, \ldots, P\}$

$$y_{\text{BB}}(n) = \sum_{p \in O(P)} \sum_{k_1=0}^{M-1} \cdots \sum_{k_p=0}^{M-1} h_p(k_1, \cdots, k_p) \prod_{\ell=1}^{\lceil p/2 \rceil} x(n-k_\ell) \prod_{\hat{\ell}=\lceil p/2 \rceil+1}^{p} x^*(n-k_{\hat{\ell}}).$$

We used the CCS architecture to derive a nonlinear digital predistorter for a model of a Q-band power amplifier (PA) whose response to a communication signal is shown in Figure 2. Note the asymmetry in the spectral regrowth to the left and the right of the signal. This suggests that multidimensional filtering will be advantageous. As demonstrated in the figure, our NDP achieves 15 dB greater ACI improvement than an LUT.
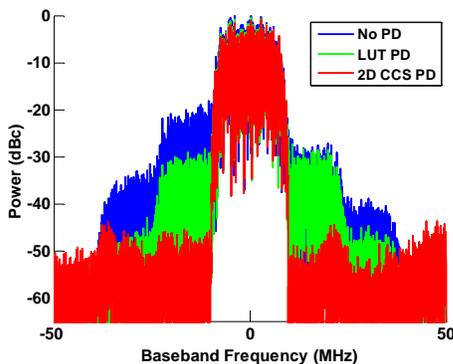
Figure 2: Q-band PA Output

We derived NDPs for this PA using various architectures: two HCS/DCS architecture, one with 4 coefficients and one with 7 coefficients per component; 2- and 3-dimensional CCS/CCS-D architectures, each with memory depth of 2 samples in each dimension (4 and 8 coefficients per component, respectively); and a "coordinate-system agnostic" architecture, where at each iteration we select a single coefficient from the Volterra kernel. We added components until the NDP achieved at least 28 dB improvement in mean square error (after which performance starts to level off). The results of this experiment are shown in Table 1. Due to the asymmetry in the nonlinear response, multidimensional filtering has a positive impact. For a comparable number of coefficients per component, the CCS/CCS-D architectures outperform HCS/DCS. While we can achieve the same amount of improvement with fewer coefficients, as we see in the "agnostic" case, regularized components allow us to exploit closeness in the kernel and avoid redundant computations, thus reducing the real-time complexity.

Table 1: Computational Requirements to achieve 28 dB Improvement in MSE

Architecture	Operations Per Sample	Number of Components
Agnostic	77 Mult., 14 Add.	15
7-coef. HCS/DCS	92 Mult., 55 Add.	8
3-D CCS/CCS-D	66 Mult., 31 Add.	4
4-coef. HCS/DCS	72 Mult., 35 Add.	9
2-D CCS/CCS-D	47 Mult., 19 Add.	5

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