Parallel FFT and Parallel Cyclic Convolution Algorithms with Regular Structures and no Processor Intercommunication

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Approach

- Perform a one dimensional Cyclic Convolution using parallel subconvolutions based on the use of Block Pseudo-Circulant Matrices.
- If a DFT is needed, perform the DFT through a Cyclic Convolution.
- Block Pseudo Circulant Matrices arise from the multiplication of Circulant Matrices by Stride_Permutations.
- Pre-processing and post-processing stages are needed to compensate for the multiplication by Stride Permutations.
- The length of the sequences, N, should be composite such as: $N=RS$, $N=R^M$ or others. There is no need for the factors to be mutually prime.
- Further restrictions are not imposed.

Basic Relation that Lead to Block Pseudocirculant Matrices in the Contex of Cyclic Convolution

$$y = H_N x$$

Example: R=4

$$Yp = P_{N,4} y = P_{N,4} H_N (P_{N,4}^{-1} P_{N,4}) x$$

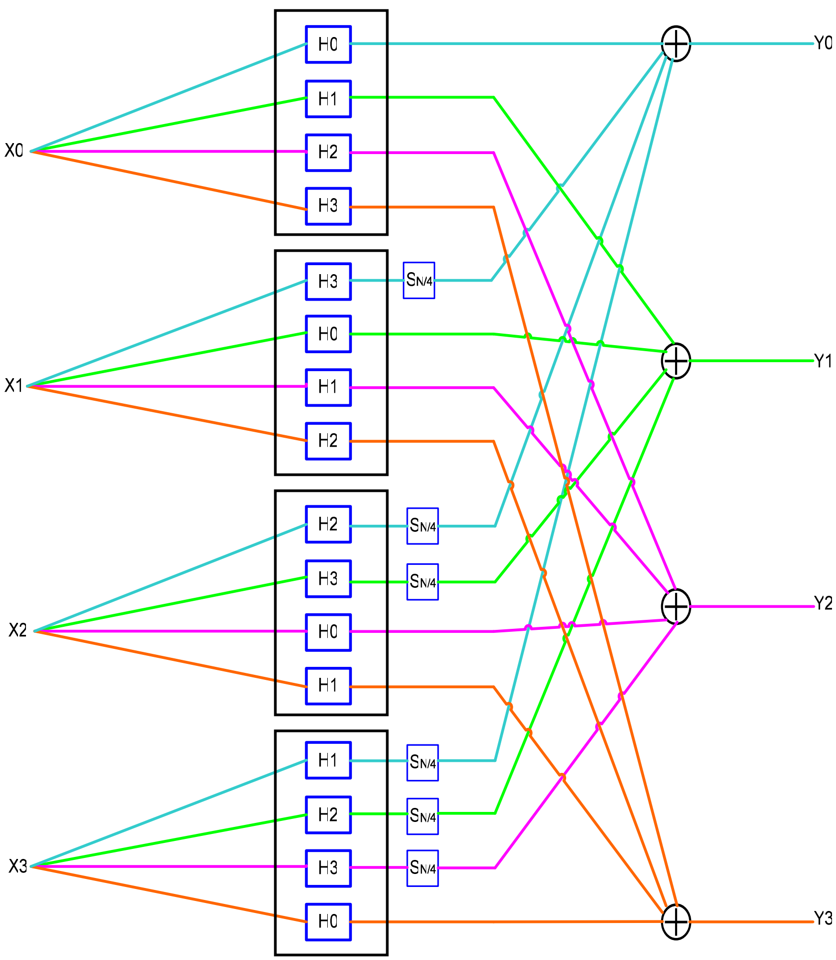
$$Yp = P_{N,4} y = (P_{N,4} H_N P_{N,4}^{-1}) P_{N,4} x = (H_P) P_{N,4} x$$

Where H_N is a Circulant Matrix and H_p is a Block Pseudo Circulant matrix

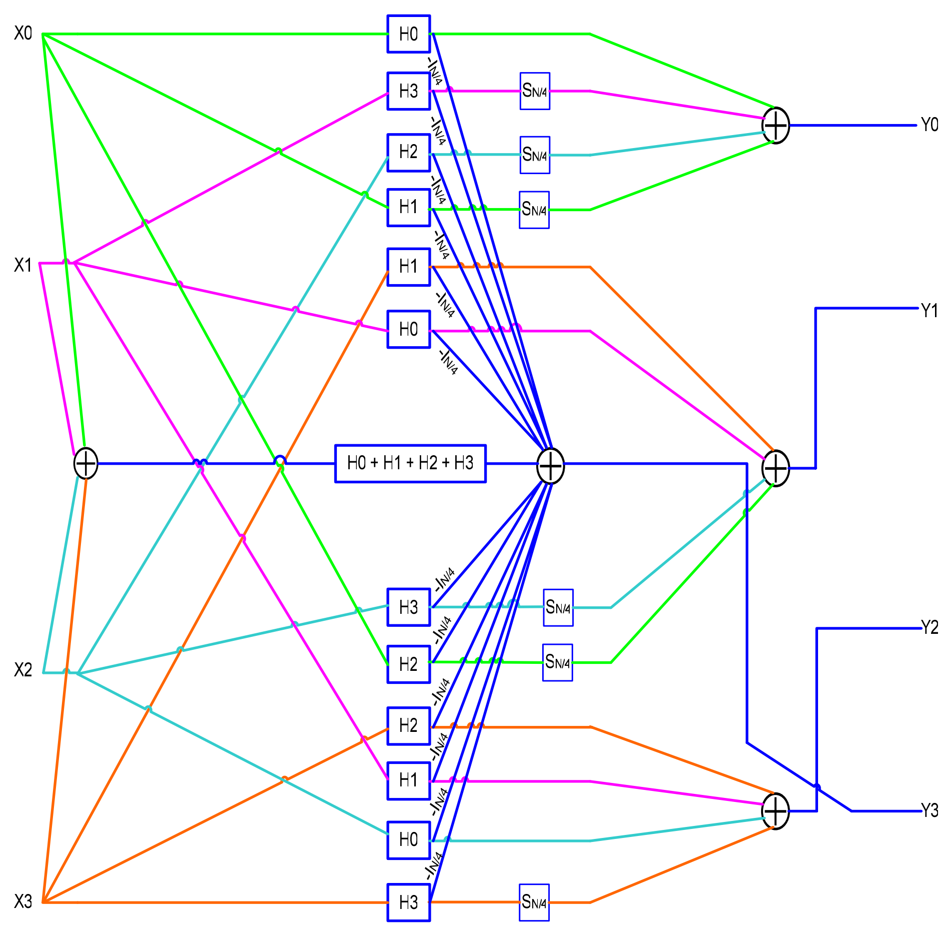
$$Yp = P_{N,4} y_n = Y_P = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} H_0 & S_{N/4} H_3 & S_{N/4} H_2 & S_{N/4} H_1 \\ H_1 & H_0 & S_{N/4} H_3 & S_{N/4} H_2 \\ H_2 & H_1 & H_0 & S_{N/4} H_3 \\ H_3 & H_2 & H_1 & H_0 \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \end{bmatrix} = (H_P) P_{N,4} x$$

Where $S_{N/4}$ is a Cyclic Shift Operator

Because of the Block Pseudocirculant Matrices the Cyclic Convolution can now be Computed using Parallel Subconvolutions of Length N/R



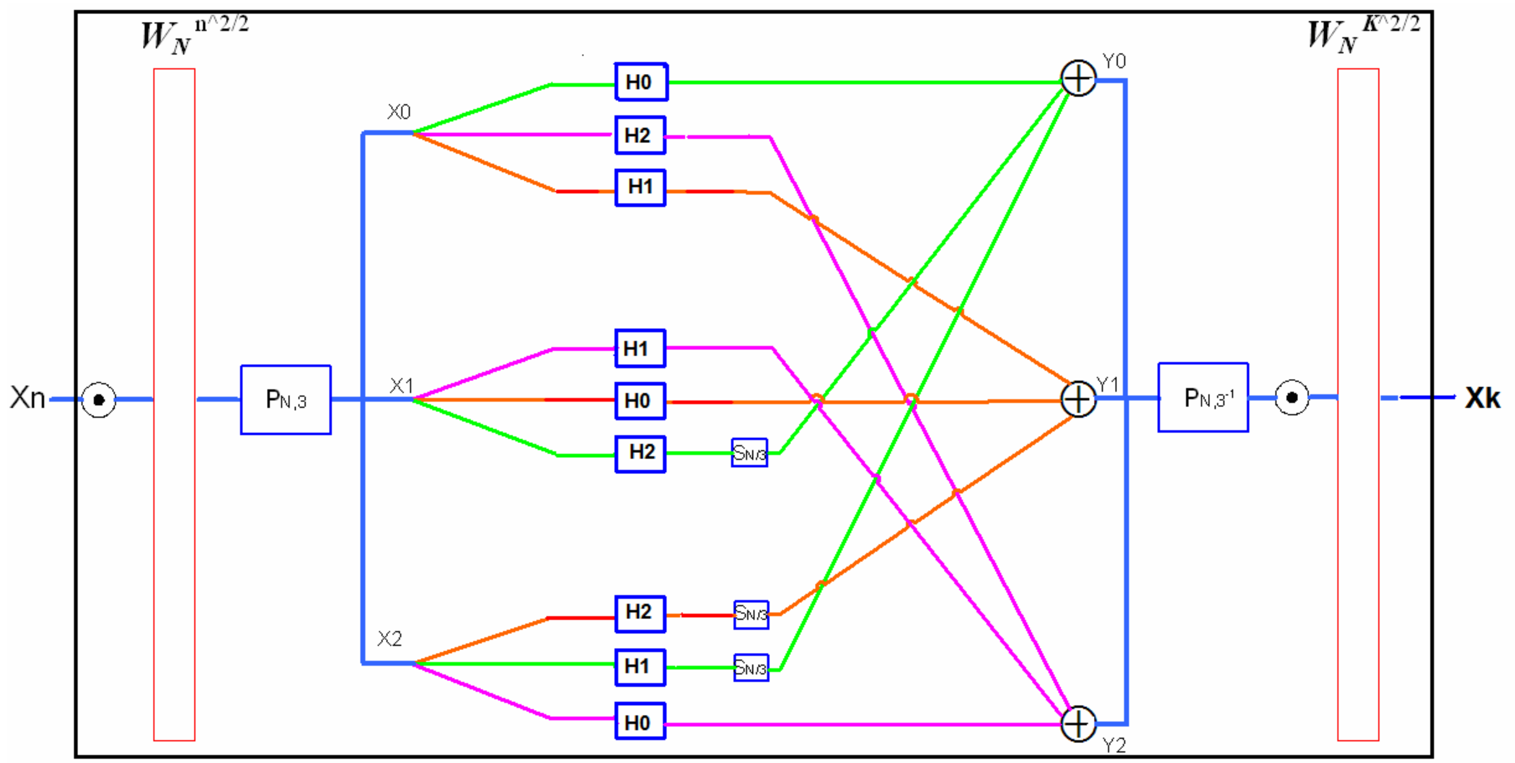
Direct Realization for R=4



Realization for R=4 after Factorization of H_p

Parallel DFT using parallel Cyclic Subconvolutions

R=3



N-Point DFT

The Bluestein algorithm can be used to write a DFT as a Cyclic Convolution

$$X[k] = W_N^{\frac{k^2}{2}} \sum_{n=0}^{N-1} (W_N^{\frac{n^2}{2}} x[n]) W_N^{\frac{-(k-n)^2}{2}} = W_N^{\frac{k^2}{2}} \sum_{n=0}^{N-1} (x_{new}[n]) h[n-k]$$