# Time Frequency Analysis for Single Channel Applications

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*The Ultimate Performance Machine*

# Project Description

## Implementation/Demonstration Goals

- Choose a selection of compute-intensive signal processing algorithms for demonstration on a real-time multicomputer system
- Some algorithms address problems in signal intercept or passive/active radar applications
- Follow progress of an interesting series of works performed at Naval Postgraduate School [2] (under Prof M. Fargues and former Prof R. Hippenstiel); also follow Time-Frequency toolbox [6].
  - **Spectral Correlation Receiver based upon FFT Accumulation Method**
  - **Continuous Wavelet Transform (Scalogram)**
  - **Discrete Wigner-Ville Distribution with a selected set of interference-reducing kernels**
  - **Parallel Filter Bank and Higher Order Statistics detection**
    - -- **Third order cumulant detector/estimator**

# Project Description

## Demonstration System

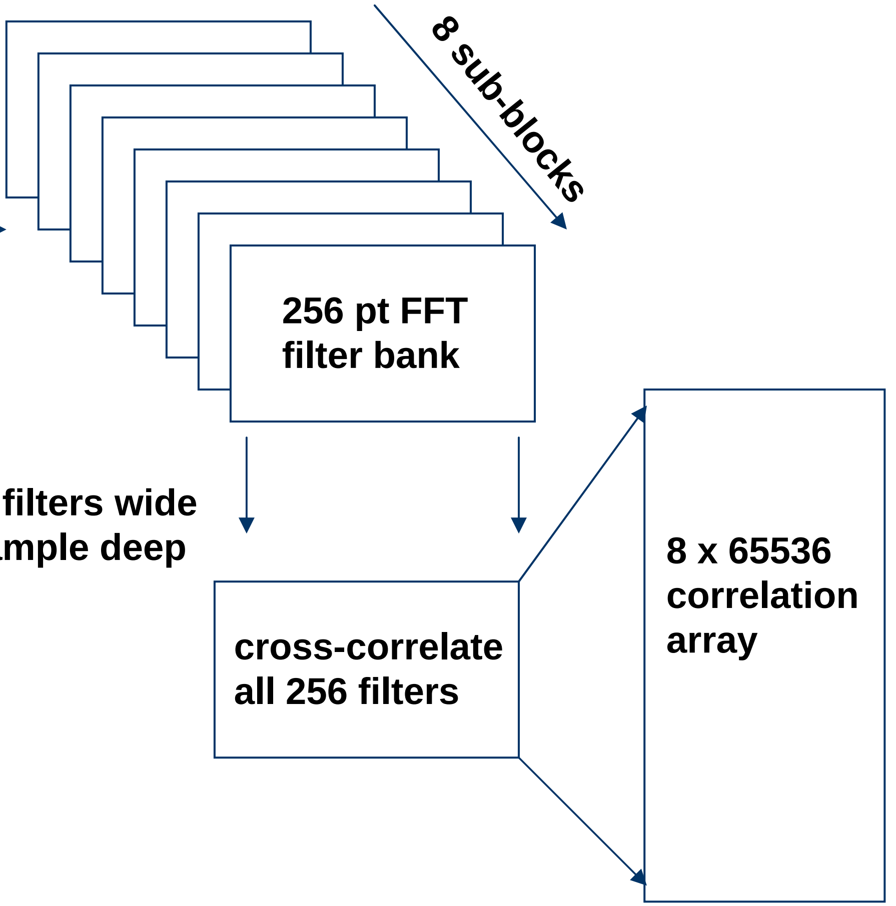
- **Common thread with all algorithms is a high-computational load distributed over multiple nodes to achieve real-time performance.**
- **Generally, a demonstration of these techniques runs on a single processor system and involves a fixed signal segment and a waiting period before presentation of results.**
- **Our contribution is to show these algorithms running in a “dynamic spectrum analyzer” mode with streaming input signal data.**
- **Near real-time graphic software written to display mesh and image plots. In addition, goal is to produce real-time contour plots.**
- **Show ease of implementation of using scientific algorithm library (SAL) library calls.**

# Time Frequency Representation (TFR) Overview

- TFRs are powerful tools to analyze, characterize, and classify dynamic signals existing in non-stationary conditions.
- Certain characteristics such as high resolution measurement of the instantaneous frequency and energy of a signal across time are appealing to practitioners across a wide range of science and engineering disciplines.
- Unfortunately the holy grail of high resolution and co-existence of multiple signals and multiple signal components remains elusive.
- An enormous amount of research focus has gone into obtaining the desirable mathematical properties of the Wigner-Ville Distribution without its accompanying distortion properties for the above conditions.
- Variety of algorithms, kernels, representations, etc. available.
- Many approaches involve high levels of computation, especially the fixes overlaid to overcome deficiencies of a particular technique.

# Spectral Correlation

## FFT Accumulation Method [4,5]



$\{ x_0, x_1, x_2, \ldots, x_{4095} \}$

4096 pt complex sample vector

8 sub-blocks

256 pt FFT filter bank

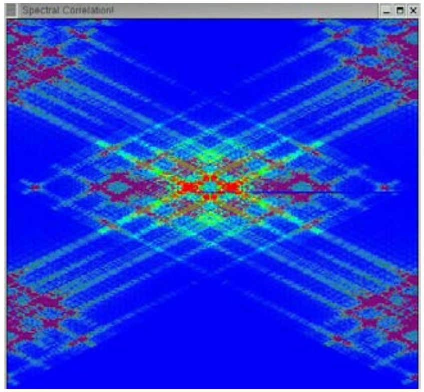
256 filters wide
8 sample deep

cross-correlate all 256 filters

8 x 65536 correlation array

reorder by cycle frequency vs. frequency

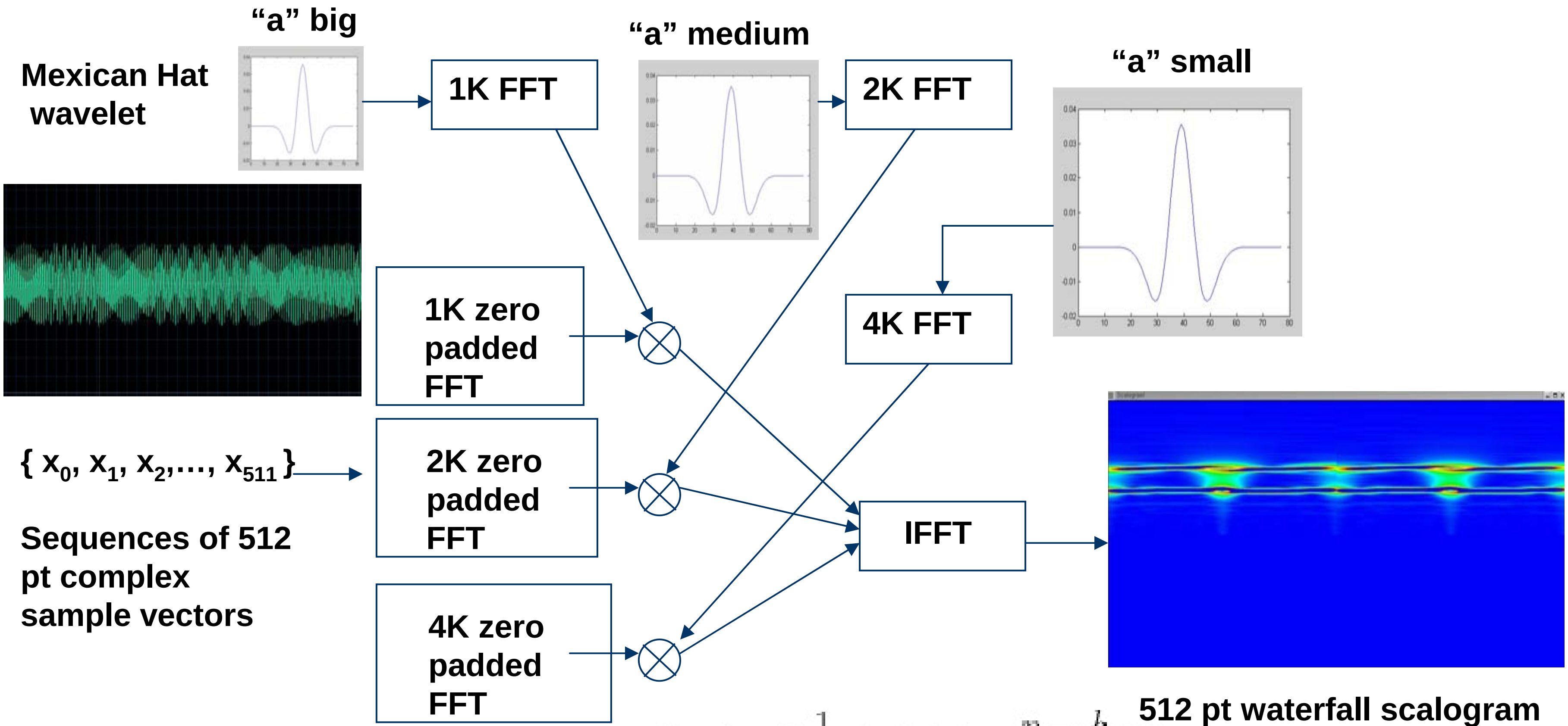
256x1024 ROS



# Scalogram (CWT)

## Continuous Wavelet Transform using fast convolution [6]

- As freq = 0.05 to 0.5, “a” scales from 10 to 1
- Wavelet basis is Mexican Hat function
- As a scales, the filter size scales logarithmically from 2263 to 47 pts
- Convolve with signal using either 4K, 2K, 1K, or 512 pt FFT



“a” big

“a” medium

“a” small

Mexican Hat wavelet

1K FFT

2K FFT

4K FFT

1K zero padded FFT

2K zero padded FFT

4K zero padded FFT

$\{ x_0, x_1, x_2, \ldots, x_{511} \}$

Sequences of 512 pt complex sample vectors

IFFT

512 pt waterfall scalogram

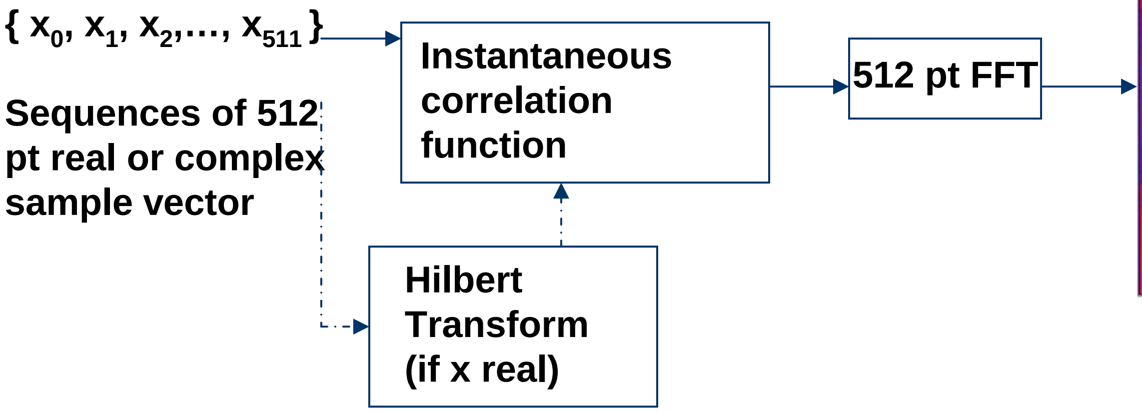
$$CWT[a,b] = \left(\frac{1}{\sqrt{a}}\sum_n s[n]\Psi\left(\frac{n-b}{a}\right)\right)^2$$

# Wigner-Ville Distribution

## Wigner-Ville Distribution [7]

- **Computed at input sample rate which drives complexity requirement**
- **Best time-frequency resolution for estimating frequencies, chirp or drift rates, event times**
- **ICF function generates interference which limits usability**
- **Satisfies many mathematical properties including energy, time and frequency marginals, instantaneous frequency and group delay**

$$WVD[m,k] = \sum_{n} s[m+n]s^{*}[m-n]e^{-j4\pi nk}$$



**$\{ x_0, x_1, x_2, \ldots, x_{511} \}$**

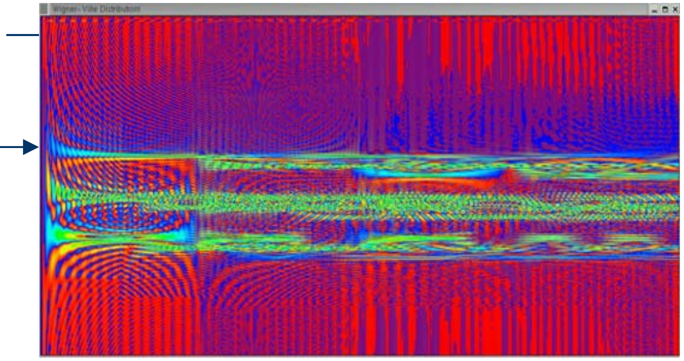
**Sequences of 512 pt real or complex sample vector**

**Instantaneous correlation function**

**512 pt FFT**

**Hilbert Transform (if x real)**

**512 pt Waterfall WVD display**



# Smoothed Pseudo Wigner-Ville Distributio

## One of many interference reduction strategies applied to WVD

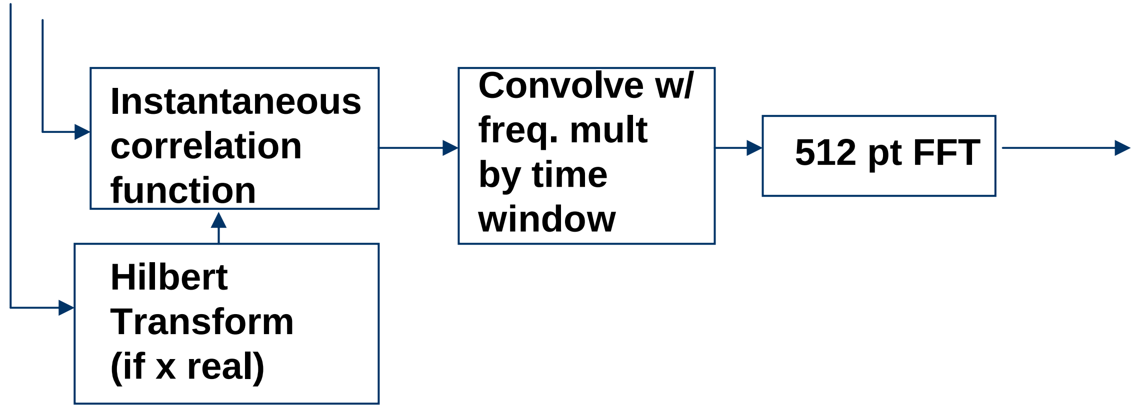
- **Time window the input sequence to suppress cross term interference. Little effect upon computation.**
- **Window in the frequency domain (convolve in time domain) which adds a significant amount to the computational complexity.**
- **Net effect is loss of resolution in time and frequency for suppression of interference.**
- **Sample rate reduction possible due to bandwidth reduction by filtering.**

**$\{ x_0, x_1, x_2, \ldots, x_{511} \}$**

$$SPWVD[m,k] = \sum_n h[n]\Big(\sum_l g[l](s[m+n-l]s^*[m-n-l]\Big)e^{-j4\pi nk}$$

**Sequences of 512 pt real or complex sample vector**

**512 pt Waterfall Smoothed Pseudo WVD**

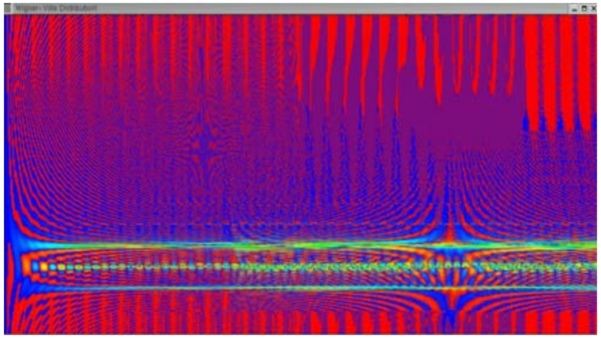


**Instantaneous correlation function**

**Hilbert Transform (if x real)**

**Convolve w/ freq. mult by time window**

**512 pt FFT**

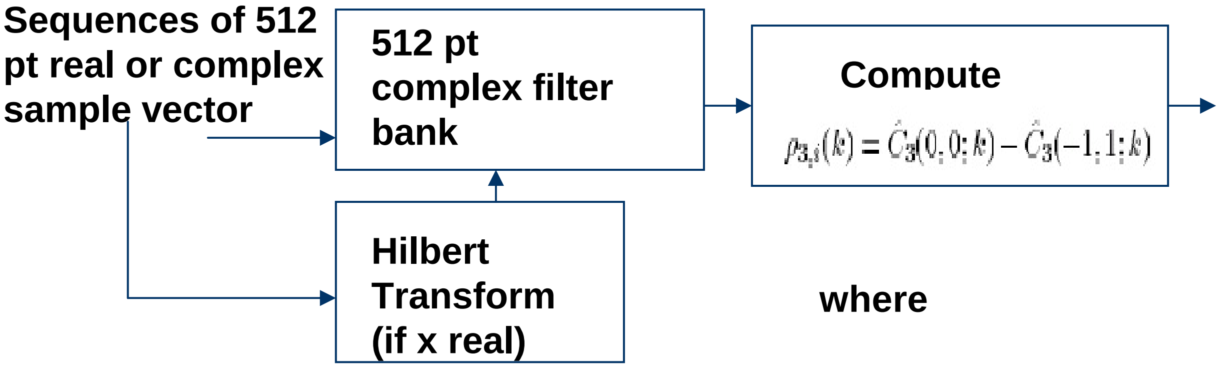


# Filterbank Spectrum with Cumulant Noise Suppression

## Time Frequency Detection Technique for Transients in Unknown Noise

- **Purpose is to demonstrate use of cumulant calculation in a real-time signal processing application.**
- **Follows work of [4]Satter,F. and Salomonsson,G. "On Detection Using Filter Banks and Higher Order Statistics," IEEE Trans. AES, Vol. 36, No. 4, Oct. 2000. Also see Taboada's report [5].**
- **Computational complexity, although relatively high, is reduced by using cumulant slices along diagonal.**
- **Based upon difference between (0,0) lag and diagonal along (-1,1) lag.**
- **Suboptimal for detection of transient low SNR signals in colored noise.**
- **Sattar, et al., derives expression of detector in terms of Teager-Kaiser energy operator and 3rd harmonic suppression.**

**$\{ x_0, x_1, x_2, \ldots, x_{511} \}$**



**Sequences of 512 pt real or complex sample vector**

**512 pt complex filter bank**

**Hilbert Transform (if x real)**

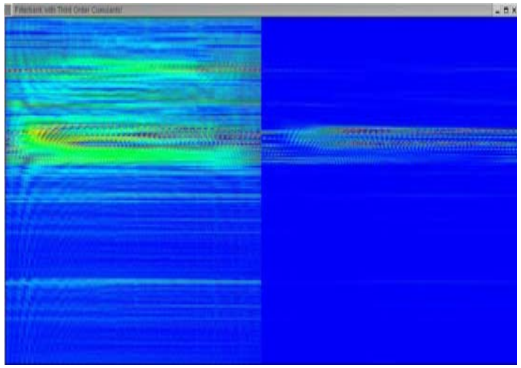
**Compute**

$$\rho_{3,i}(k) = \hat{C}_3(0,0;k) - \hat{C}_3(-1,1;k)$$

**where**

$$\hat{C}_{3,z}(l_1,l_2;k) = \sum_{n=S_1}^{S_2} z_k(n) z_k(n+l) z_k(n+l_2)$$

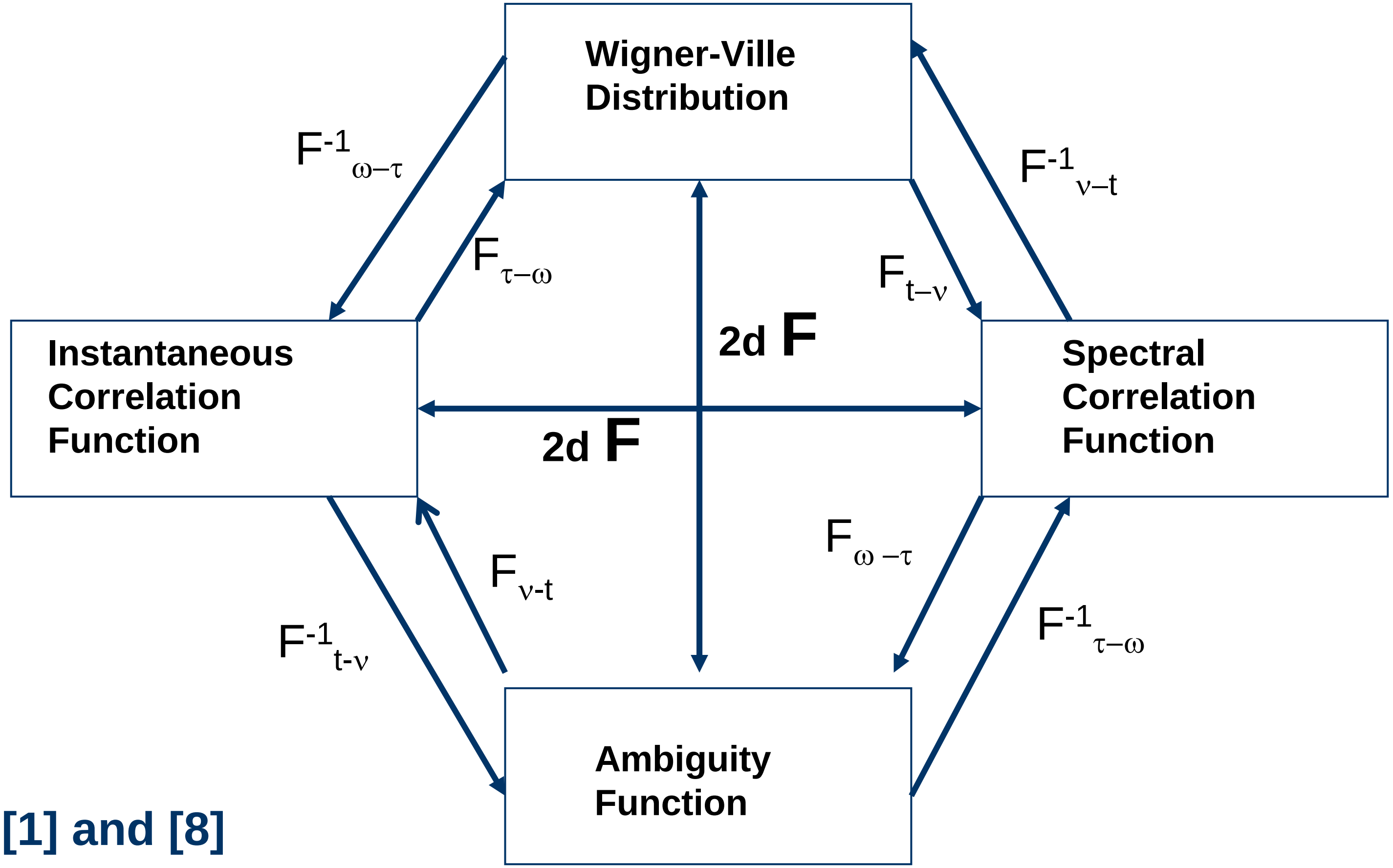
**512 pt waterfall filterbank with cumulant processing**



**pre-filter** **post-filter**

# Demonstration Algorithms

Unifying Fourier Transform relationships between demonstration algorithms



**Wigner-Ville Distribution**

$F^{-1}_{\omega-\tau}$

$F_{\tau-\omega}$

$F^{-1}_{\nu-t}$

$F_{t-\nu}$

**Instantaneous Correlation Function**

**2d $F$**

**2d $F$**

**Spectral Correlation Function**

$F_{\nu\text{-}t}$

$F_{\omega-\tau}$

$F^{-1}_{t\text{-}\nu}$

$F^{-1}_{\tau-\omega}$

**Ambiguity Function**

**See [1] and [8]**

# Time-Frequency Algorithms

## Several (non-exclusive) categorizations of T-F algorithms

### Order:

Linear
Quadratic
Hyperbolic
Power

### Invariance property:

Time/frequency shift (Cohen's) -> kernel type
Time/scale (affine)

### Signal dependence:

Signal independent
Signal adaptive

### Representation / Atomic Decomposition:

Orthogonal basis functions
Non-orthogonal elementary functions

### Mathematical Interpretation:

Physical: Complex exponentials as eigenfunction solutions
Statistical: no structural assumptions; "dictionary of tiled wavelets"

### Algorithm:

Spectrogram
Multi-windowed spectrogram
Gabor representation
Scalogram (CWT)
Discrete Wavelet Transform
Wigner-Ville Distribution
Pseudo Wigner-Ville Distribution
Smoothed Pseudo Wigner-Ville
Choi-Williams
Cone-shaped
Rihaczek
Margeneau-Hill
Page
Born-Jordan
Reassignment techniques
I/0 kernel
Radially Gaussian Kernel
Adaptive Gabor Expansion
Adaptive chirplet Decomposition
Matching Pursuit
Basis Pursuit

# Qualifications on Performance Data

- No attempt was made to lower sample rate on smoothed pseudo Wigner-Ville Distribution as made possible by filtering operations.
- No attempt has been made to optimize performance with respect to algorithmic breakdown beyond a top level.
- Example: WVD should be real, therefore could compute 2 FFT at once using odd and even input symmetries.
- No attempt has been made at optimizing performance with respect to machine and system architecture, i.e., stripmining.
- Example: Segment data blocks in consideration of processor L1 cache size to achieve fast throughput. Re-use of most recently used data segments.
- Display update rate limited by trying to get 512 KByte images through Ethernet pipe and router.

# Single Processor Measurements

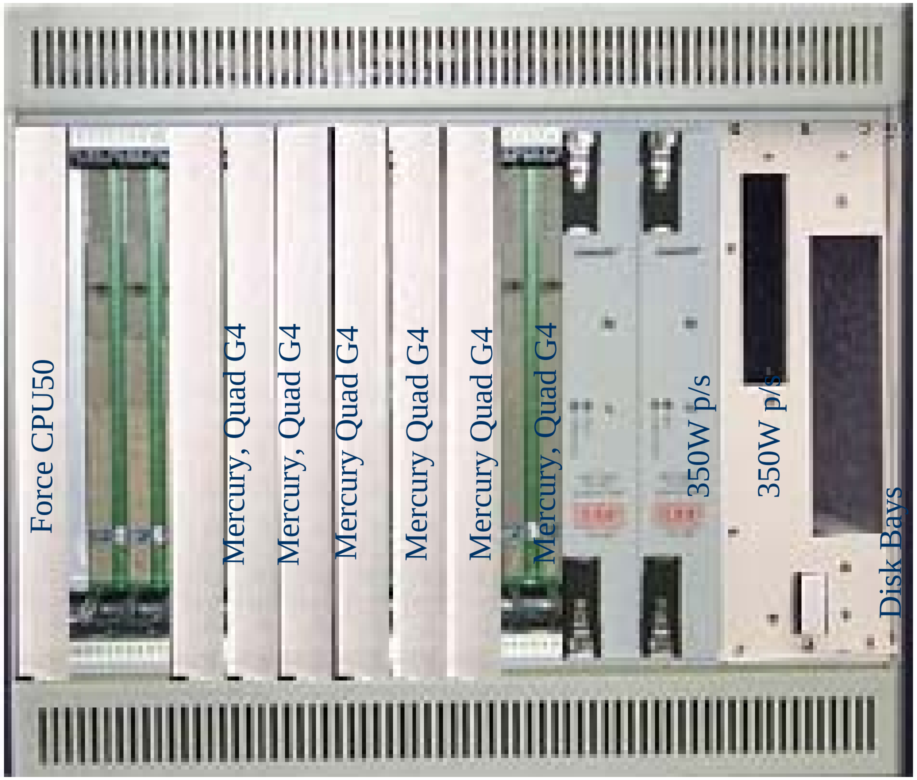
- WVD: 29 msec per 512 samples
- PWVD: 29 msec per 512 samples
- SPWVD: 650 msec per 512 samples
- Spectral Correlation: 33 msec for block of 4096 samples
- HOS filter bank: 732 msec for block of 512 samples
- Scalogram:  102 msec for block of 512 samples

**Exercise:**

**As hypothetical example, using 64 kHz sample rate, 512 samples are collected in 8 milliseconds, 4096 samples are collected in 64 milliseconds.**

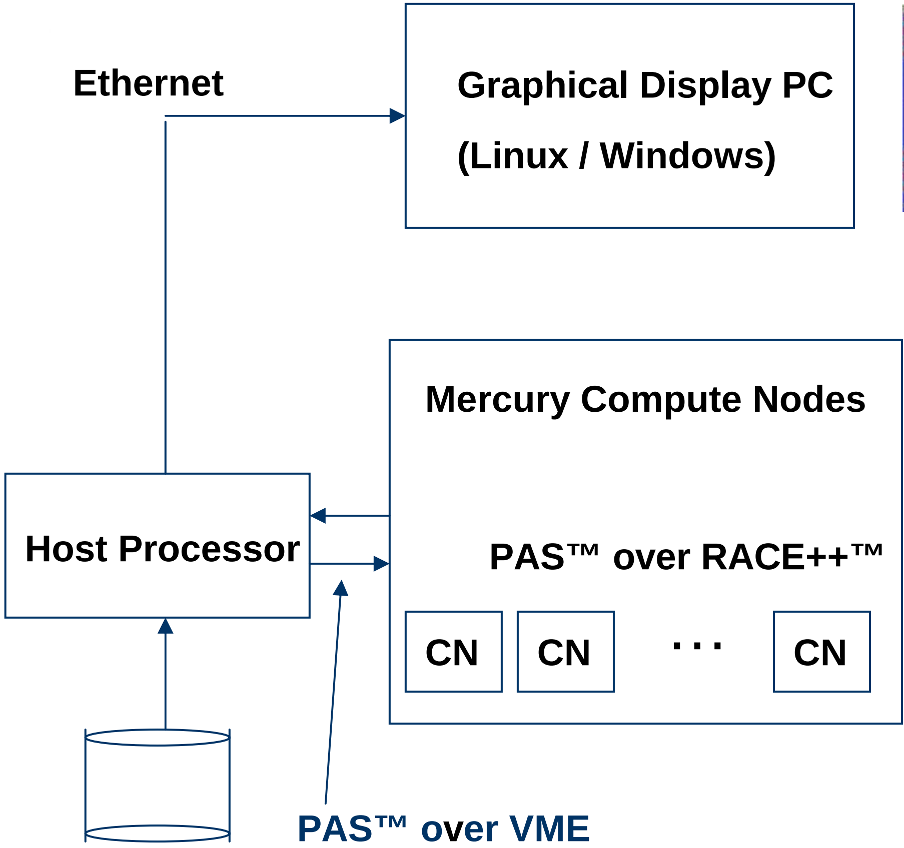
| Algorithm | Number processors |
|---|---|
| Spectral correlation | 1 |
| WVD | 5 |
| Scalogram | 12 |
| SPWVD | |
| HOS filterbank | large |

# Lab Development System



- **1X Force CPU50, 333MHz SPARC**
- **6x Mercury, MCJ6 with 4x G4 7400@400MHz, with 64Mbyte RAM each**
- **Total of 76 Gflops peak processing**
- **Total of 152Gops peak 16Bit**
- **Dual RACE++**
- **Total bisection bandwidth of 1 Gbyte/sec**

# Demo System Configuration



Ethernet

Graphical Display PC

(Linux / Windows)

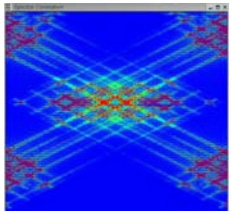
Mercury Compute Nodes

PAS™ over RACE++™

CN CN . . . CN

Host Processor

PAS™ over VME



512x512 image

Data Files

**Software Tools Used**

Mercury SAL™ (Scientific Application Library)

Mercury PAS™ (Parallel Acceleration System)

SDML (Simple Direct Media Layer)

SGE (SDML Graphics Extension)

SDL Draw

# Selected References

1. Taboada,F., “Detection and Classification of LPI Radar Signals Using Parallel Filter Arrays and Higher Order Statistics,” Sept. 2002 Thesis.
2. Taboada, F., Lima, A., Gau, J.Jarpe,P. Pace, P. , “Intercept Receiver Signal Processing Techniques to Detect LPI Radar Signals,” ICASSP, 2002.
3. Satter,F. and Salomonsson,G. “On Detection Using Filter Banks and Higher Order Statistics,” IEEE Trans. AES, Vol 36, No. 4, Oct. 2000.
4. Gardner, W., “Exploitation of Spectral Redundancy in Cyclostationary Signals,” IEEE Signal Processing Magazine, Vol. 8, No. 2, pp.14-32, April 1991.
5. Roberts, R., Brown, R., Loomis, H., “Computationally Efficient Algorithms for Cyclic Spectral Analysis”, IEEE Signal Processing Magazine, Vol. 8, No. 2, pp. 38-49, April 1991.
6. Time-Frequency Toolbox, Version 1.0, January 1996, Copyright (c) 1994-96 by CNRS (France) - RICE University (USA).
7. Qian, S., “Introduction to Time-Frequency and Wavelet Transforms,” Prentice Hall PTR, Upper Saddle River, NJ, 2002.
8. Debnath,L, ed., “Wavelet Transforms and Time-Frequency Signal Analysis,” Birkhauser Boston, New York, NY, 2001.