New FFTW Developments

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FFTW

- C library for computing discrete Fourier transforms.
- Arbitrary size, multiple dimensions, real and complex.
- Widespread use. Ships with Matlab 6.
- Unusual implementation:
  - Adapts to hardware. Portability and high performance at the same time.
  - Computational kernels (95% of the code) generated automatically.
- Written by Matteo Frigo and Steven G. Johnson. Contributions from Franz Franchetti, Stefan Kral. Support from MIT, Lincoln Labs, TU Wien, DARPA, NSF, AMD, DEC, Intel, Sun.

FFTW version 3

- Released in April 2003.
- Available at `http://www.fftw.org/`, GPL.
- Fourier, real Fourier, cosine, sine, and Hartley transforms.
- Support for *interleaved* and *split* complex arrays.
- *SIMD* (SSE, SSE2, 3DNow!, Altivec) support, even for sizes $\neq 2^k$.
- Fused multiply-add support (PowerPC, IA-64, ...).
- Faster out-of-cache transforms.
- Improved accuracy.

Technical innovations in FFTW3

- New flexible infrastructure for automatic adaptation. Experimental FFTW system for convolutions.
- New SIMD complex-FFT algorithm.
- Automatic parallelizer extracts 2-way SIMD parallelism from sequential FFT programs. [Kral]
- New algorithm for real transforms of prime size, based upon [Rader 1968].
- New optimizer derives DCT/DST algorithms automatically from an FFT algorithm.
- Portable SIMD framework. Same C code works on SSE, SSE2, 3DNow!, Altivec. [Franchetti]

Outline

- Benchmarks.
- Structure of FFTW3.
- How FFTW3 uses SIMD instructions:
  - The complex-FFT SIMD algorithm.
  - The automatic vectorizer.
- Conclusions.

Benchmarks

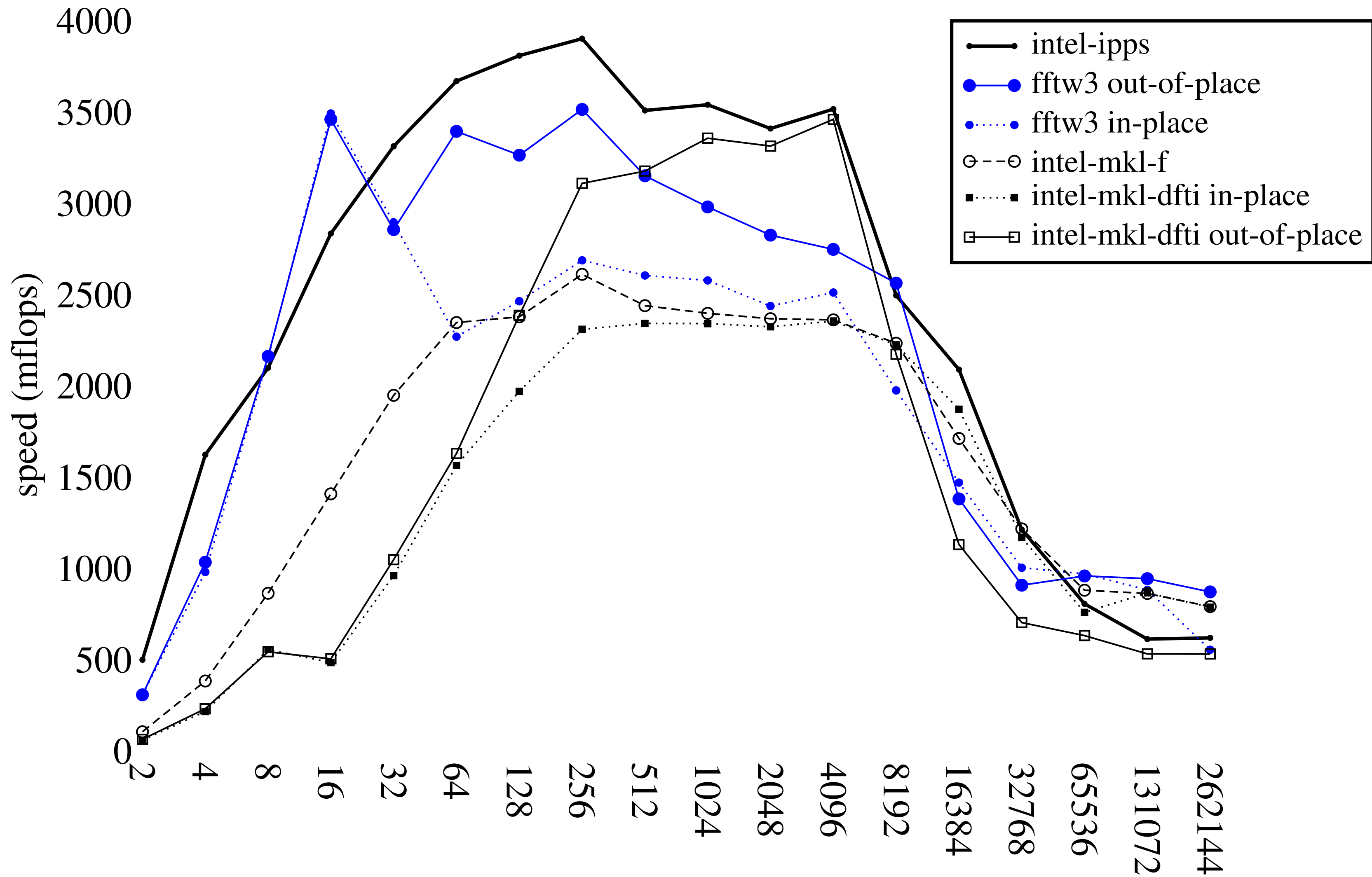
Next four slides:

- FFTW3 vs. Intel MKL/IPPS.
- double precision, SSE2.
- 2.8 GHz Pentium IV, `icc -O3`, 256 KB L2.
- fftw-3.0.1, MKL 6.0, IPPS 3.0, icc 7.1.
- "mflops" := $5N \log N \;/\;$ time in μs.

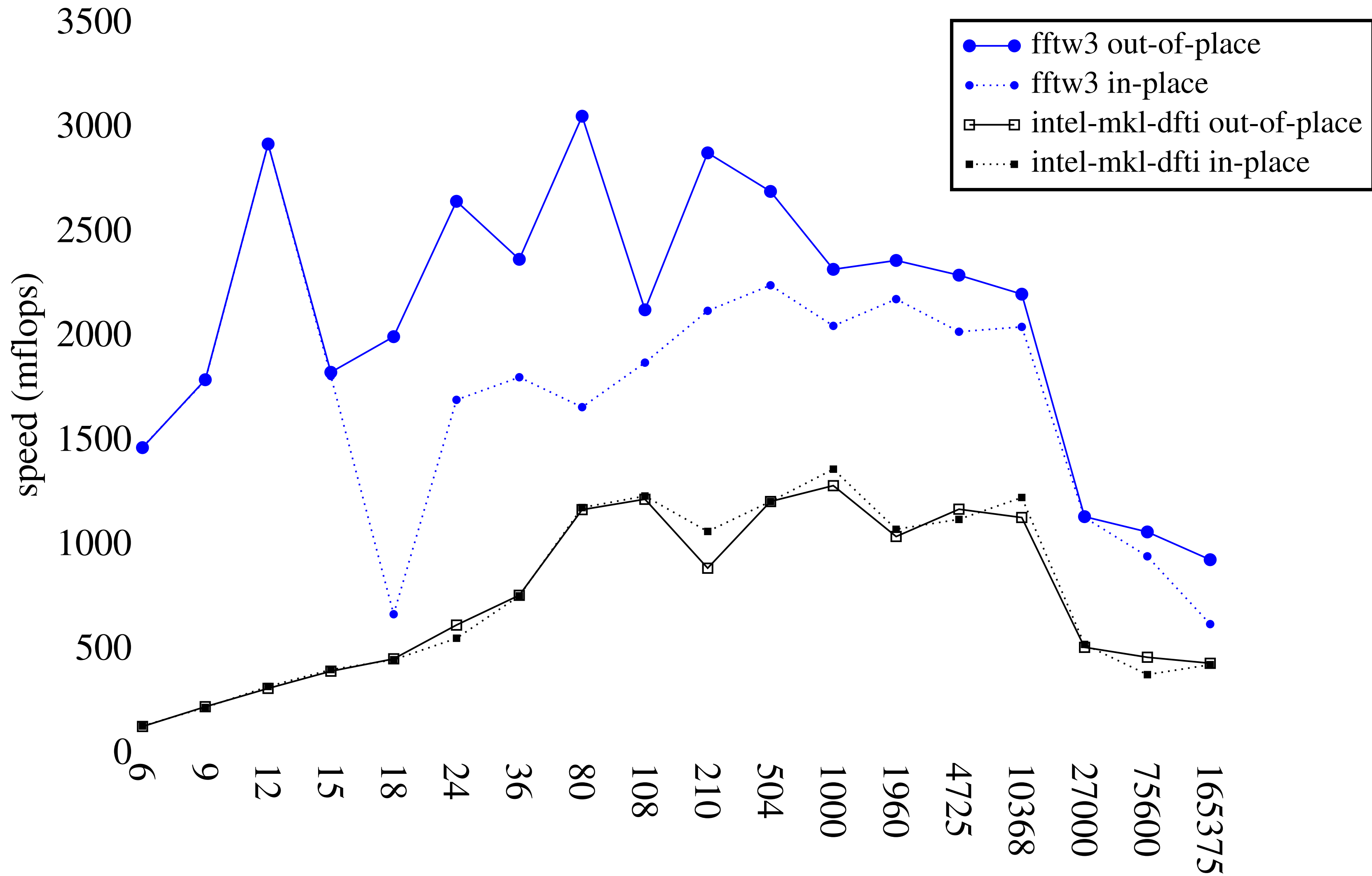
Many more benchmarks at

`http://www.fftw.org/`

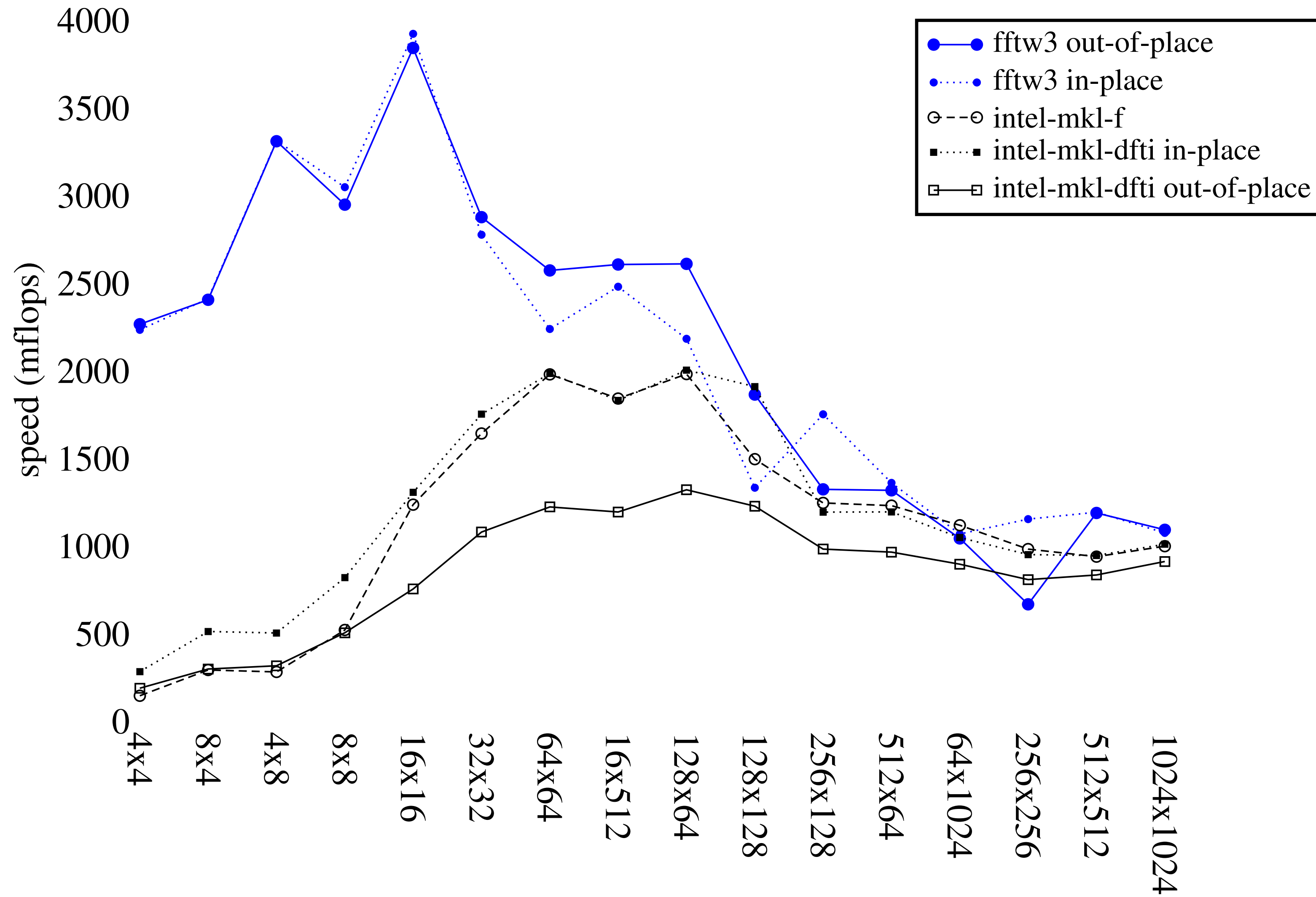
FFTW3 vs. Intel, 1D, 2^k



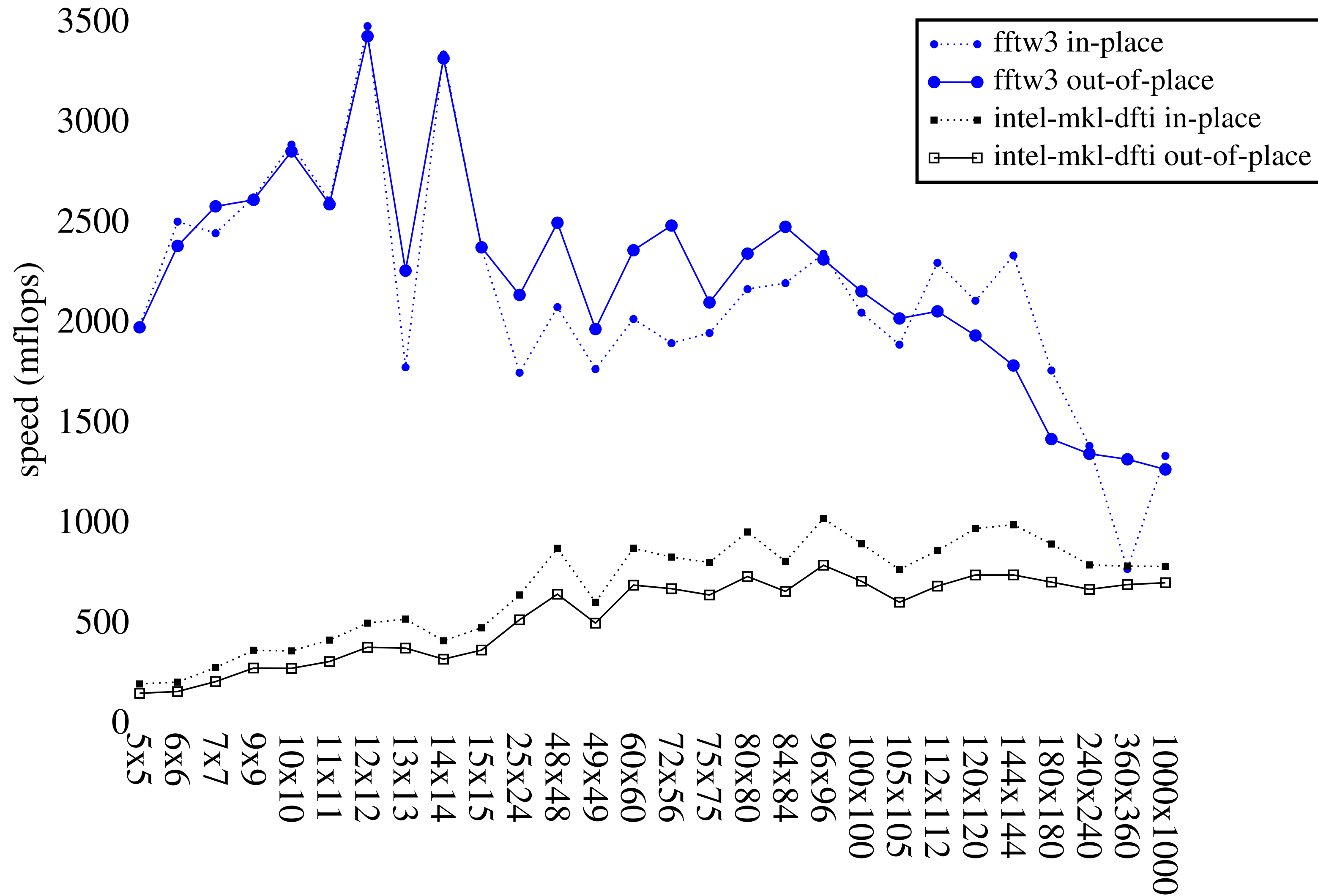
FFTW3 vs. Intel, 1D



FFTW3 vs. Intel, 2D, 2^k



FFTW3 vs. Intel, 2D



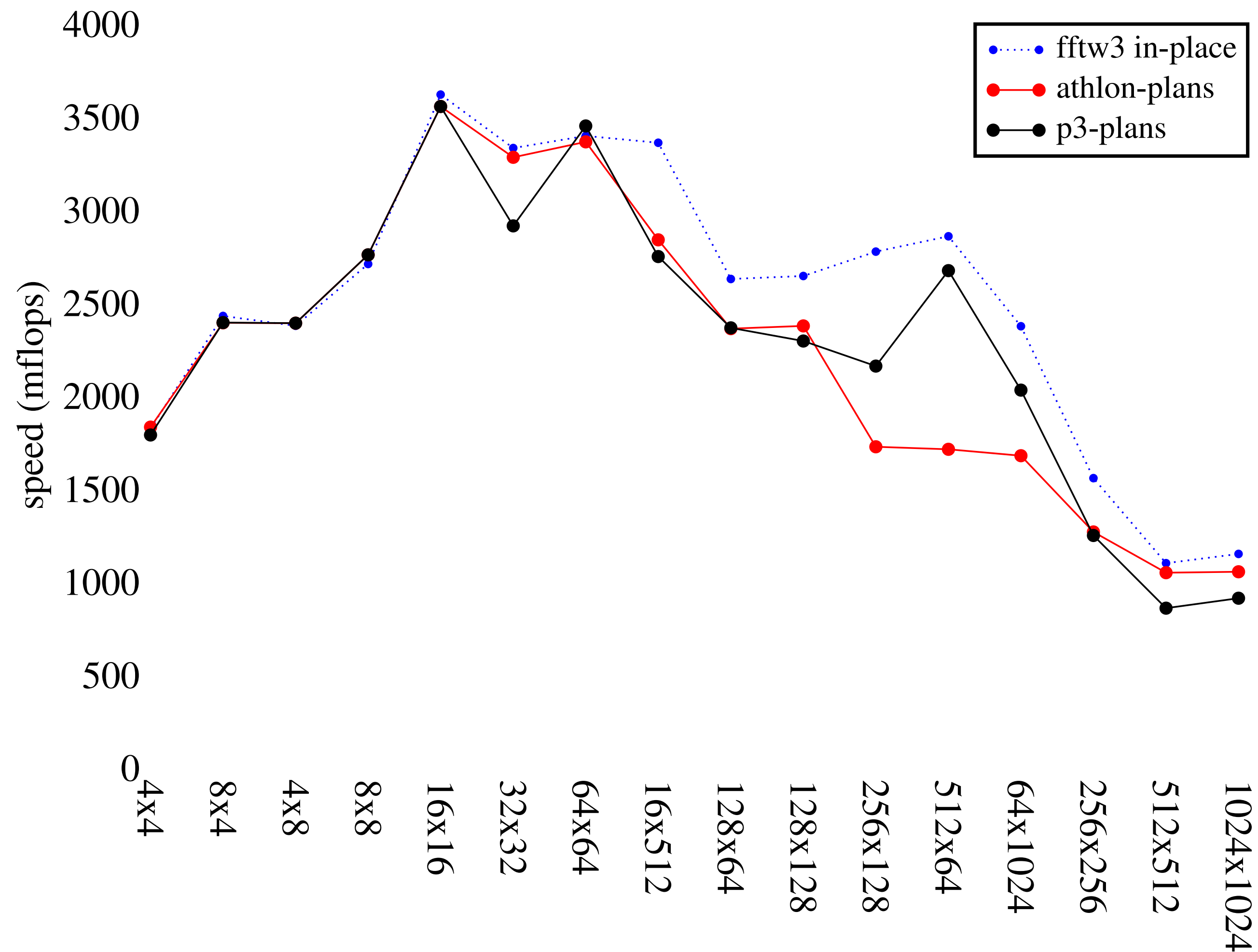
Structure of FFTW3

- A ***problem*** describes a computation to be performed.
  - E.g., 7 forward complex DFTs of size 13.
- A ***plan*** performs the computation described by a problem.
  - Most of the code for plans is generated automatically.
- A ***solver*** looks at the problem and it either fails or returns a plan.
  - E.g.: radix-r Cooley-Tukey solver, for various r.
  - E.g.: multidimensional solver reduces d-dimensional problems to 1D problems.
- Given problem, the ***planner*** tries all applicable solvers and picks the fastest.
  - Various heuristics to speed up the search.

FFTW3 is flexible

- The planner knows nothing about FFTs.
  - Same planner works for FFT, DCT, convolution, etc.
- To try a new algorithm/trick, just add a solver.
  - Decimation in time or decimation in frequency?
  - Compute twiddle factors or load them from memory?
  - Copy the input into a buffer for better locality?
  - Best order in d-dimensional transforms?

Why a planner?



FFTW3 on 2.2 GHz Pentium IV vs. FFTW3 with plans from Pentium III, Athlon.

FFTW's SIMD strategy

Two competing approaches:

1. Special SIMD FFT algorithm.
   - Semi-portable C code.
   - Works for 2-way and 4-way SIMD.
   - Complex transforms only.
2. Automatic vectorizer.
   - Generates machine-specific assembly.
   - Works for 2-way SIMD only.
   - Currently only supports 3DNow!.

FFTW's SIMD FFT algorithm

Cooley-Tukey: r FFTs of size n/r, plus n/r FFTs of size r.

Idea: Compute the *complex* FFT of size r as a vector of two *real* FFTs of size r:

$$\text{FFT}(X) = \text{FFT}(\text{Re}(X)) + i \cdot \text{FFT}(\text{Im}(X)) \ .$$

Arithmetic complexity same as Cooley-Tukey.

Works for 2-way SIMD. For 4-way, do two steps in parallel.

Other implementations compute multiple butterflies in parallel. FFTW exploits parallelism within a butterfly.

Discussion of SIMD algorithm

Good:

- Works whenever Cooley-Tukey works, including odd sizes.
- 2-way SIMD: no need to worry about strides/alignment.
- Works for normal complex arrays—no weird data formats.
- Uses fewer registers than scalar code for same r.

Bad:

- Hard to implement, but we generate code automatically anyway.
- Extra shuffling at the end of each radix-r step.

Automatic two-way vectorizer

Example: size-3 FFT

```
r0 = in[0];
r2 = in[2];
r4 = in[4];
r6 = r2 + r4;
r8 = r4 - r2;
r10 = .866 * r8;
r12 = .5 * r6;
r14 = r0 - r12;
r16 = r0 + r6;
r18 = r14 + r11;
r20 = r14 - r11;
out[0] = r16;
out[2] = r18;
out[4] = r20;
```

```
r1 = in[1];
r3 = in[3];
r5 = in[5];
r7 = r3 + r5;
r9 = r3 - r5;
r11 = .866 * r9;
r13 = .5 * r7;
r15 = r1 - r13;
r17 = r1 + r7;
r19 = r15 + r10;
r21 = r15 - r10;
out[1] = r17;
out[3] = r19;
out[5] = r21;
```

Two-way vectorized FFT

```
(r0, r1) = (in[0], in[1]);
(r2, r3) = (in[2], in[3]);
(r4, r5) = (in[4], in[5]);
(r6, r7) = (r2, r3) + (r4, r5);
(r8, r9) = ((r2, r3) - (r4, r5)) * (-1.0, 1.0);
(r10, r11) = (.866, .866) * (r8, r9);
(r12, r13) = (.5, .5) * (r6, r7);
(r14, r15) = (r0, r1) - (r12, r13);
(r16, r17) = (r0, r1) + (r6, r7);
(r10, r11) = (r11, r10);
(r18, r19) = (r14, r15) + (r10, r11);
(r20, r21) = (r14, r15) - (r10, r11);
(out[0], out[1]) = (r16, r17);
(out[2], out[3]) = (r18, r19);
(out[4], out[5]) = (r20, r21);
```

Vectorization algorithm

Input: FFT program, given as flow graph.

Goal: pair each node with some other node.

How: brute force.

- Maintain a set of paired nodes, initially empty.
- While unpaired nodes exist:
  - Nondeterministically choose two unpaired nodes.
  - Pair them according to "pairing rules". If this step fails, backtrack.
- Perform local optimizations.

Example: pairing rule for loads

If `addr1 = addr0 + 1`, rewrite

```
v0 = mem[addr0];
v1 = mem[addr1];
```

into

```
(v0, v1) = (mem[addr0], mem[addr0 + 1]);
```

Pair `(v0, v1)`.

(SIMD load works only on consecutive addresses.)

When does the vectorizer work?

- Complex transforms of any size: OK.
  - Vectorizer usually (but not always) pairs real and imaginary part of a complex number (modulo conjugation and swap).
- Real transforms of even size: OK.
  - Not clear what the vectorizer does.
- Real transforms of odd size: FAIL.
  - Cannot pair an odd number of loads, stores, or flops.

Conclusion

FFTW3 is a flexible framework for implementing FFTs and related computations.

In progress:

- Implement SIMD real transforms.
- Convolutions.
- Large 1D transforms.